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**Research in Solar Plasma Theory**

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Department of Physics

University of California

Irvine, CA 92717

Gerard Van Hoven, Principal Investigator

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## FINAL TECHNICAL REPORT

The NASA Solar-Terrestrial-Theory program, "Research in Solar Plasma Theory" has been supported by Grant NAGW-93 at the University of California, Irvine since 1 June 1980. During the intervening years, the UCI group has produced 58 refereed publications [listed in Appendix I] and three Ph.D. dissertations (#2, #12, #43) acknowledging this grant. The results of this research effort have been described in nearly a score of Invited Talks presented at scientific meetings and workshops. In addition, we have created five numerical codes for the supercomputer simulation of solar activity phenomena, and have significantly adapted three others. [Brief descriptions of those codes are given in Appendix II.]

Since the Abstracts of all of our publications are provided in Appendix III, we will concentrate in this report on the main thrusts and significance of our research results. [The numbers refer to the publication list provided in Appendix I.] Three recent invited reviews of the UCI solar-activity simulation program (#42, #50, #56) are also available in print.

### A. CORONAL STRUCTURE AND DYNAMICS

We have spent considerable effort in trying to understand the magnetohydrodynamic (MHD) structure, evolution and stability of the myriad of coronal forms observed by space-borne telescopes.

#### A.1. Magnetic Loops

An early emphasis was to understand the MHD stability of the ubiquitous coronal loops so comprehensively observed by the wide spectrum of Skylab instruments. The fact that they persisted for days flew in the face of the well-known laboratory instability of localized, magnetically confined, dense plasma structures. Earlier work, by the UCI group and others, had suggested a qualitative stabilizing influence, magnetic line tying at the loop ends. We then set out to make the effects of line tying more realistic physically (#1) and more rigorous mathematically (#3). The end result was the ability to compute the instability boundaries, in terms of the length-to-radius aspect ratio for realistic coronal loops, and finally to understand the reason for their MHD stability (#4). A second aspect of loop behavior is their magnetic activity, when they make the transition to the resistive- reconnection process of a

compact flare. Both the linear and nonlinear (#20) energy-release aspects of such loop flares were evaluated.

We have also recently developed and applied a *three-dimensional* MHD model to the study of the *dynamical* evolution of coronal loops. This model can self-consistently represent the various phases of development of coronal loops, from the quasi-static evolution through equilibrium states as a result of twisting photospheric flow profiles, through the linear ideal and resistive MHD instability phase, and through the nonlinear behavior after kink instabilities grow to large amplitudes and cause field-line reconnection. We have used this model to study the equilibrium properties and ideal linear MHD kink stability of coronal loops (#52). We have found that typical localized-twist profiles produce loops with zero net axial current which are unstable to internal kink modes (and not to the faster-growing external kink modes). We have also confirmed and extended previous results on the stability of the idealized Gold-Hoyle model of a loop (with uniform twist). A preliminary solution of the MHD equations for the *nonlinear evolution* of the internal kink mode has shown that *concentrated current layers* develop as a result of the instability (#50). These layers have profound consequences in regard to the formation of current sheets in the line-tied solar corona, and are intimately related to the coronal heating problem.

## **A. 2. Helmet Streamers**

In connection with our mass-ejection studies, to be described in the next section, we have investigated helmet streamers, large-scale structures that are seen frequently in the solar corona. Observations indicate that coronal mass ejections (CMEs) are often associated with helmet streamers and the disruption of overlying streamers by CMEs has been reported. These observational studies have convinced us that investigating the stability of streamer-like magnetic configurations may yield important insights about CME initiation.

Previous simulation studies of streamers were two-dimensional (axisymmetry assumed). Although a two-dimensional description is useful, observations indicate that coronal streamers have a finite longitudinal extent, and are thus three-dimensional objects. The 3-D structure of streamers could be an important aspect of their stability. Using a transonic Parker wind as the initial condition for the density, pressure and velocity (as was done in previ-

ous studies) and a non-axisymmetric potential magnetic field (never previously attempted) we have performed the first *three-dimensional* simulations of coronal streamers (#58). A streamerlike configuration with a finite longitudinal extent forms. The closed field region has a higher plasma density and is bounded by a current sheet. We have also treated open and closed fields in the stellar context (#6).

### **A. 3. Mass Ejections**

An important manifestation of solar activity is the coronal mass ejection. CMEs add mass and magnetic fields to the solar wind and may cause interplanetary shock waves, which provide a causal link between solar and geomagnetic activity. We have performed several studies of the evolution of CMEs in the solar corona. As most previous simulations used a thermal driver as the mechanism for CME initiation, we have examined some of the possible effects of a non-thermal driver; specifically, plasmoid ejection. As a simple model of an erupting prominence we accelerated a cold, dense, plasma parcel to a given outward velocity, then followed the time evolution as the parcel moved through an initially hydrostatic, current-free atmosphere. We found that this simple driving mechanism yielded results that are consistent with many of the observational characteristics of CMEs, suggesting that the form of the driving mechanism in fact plays an influential role in the evolution of CMEs (#48). We have also performed studies in which we incorporated a plasmoid-like *magnetic* topology and allowed the force resulting from the current associated with this magnetic structure to drive plasma outward (#54).

## **B. CORONAL HEATING**

One of the principal unsolved problems of solar physics is the mechanism for heating the corona. We have made significant contributions to the elucidation of the Parker "topological dissipation" proposal and of the Alfvén surface-wave model.

### **B. 1. Current Filaments**

We have studied an atmospheric heating model, due to Parker (1972, 1986), in which the convective twisting of the coronal magnetic field leads to strong currents. We developed detailed solutions to the 3-D ideal-MHD equations describing the coronal response when a

sequence of smooth, quasi-random, photospheric flows was applied (#40). These solutions show that *smooth flows* characteristic of photospheric convection can *induce a fine-scale spatial structure* in the coronal magnetic field, leading to the formation of concentrated current filaments. The resistive dissipation of these currents can heat the corona (Parker 1986, van Ballegooijen 1986, 1988).

The principal results show that smooth flows can be very effective in generating current filaments in the corona. In particular, spatial structure is induced at increasingly shorter length scales, with an exponential dependence of the shortest length scale on the magnetic foot-point displacement. The spatial structure is generated through nonlinearities in the MHD equations, and cascades from the driven long wavelengths to shorter wavelengths. This mechanism could be an ingredient of a coronal heating model if the cascade of spatial structure continues to the short length scales at which the magnetic field can be dissipated resistively or by reconnection in the highly conducting corona. In future research it would be desirable to determine whether this mechanism satisfies the quantitative requirements of a coronal heating model.

## **B. 2. Alfven Surface Waves**

We have investigated the coronal-heating potential arising from the propagation and dissipation of Alfven waves in a non-uniform plasma embedded in a magnetic field with continuous transverse gradients (#27). Such configurations arise in the neighborhood of active regions on the solar surface and (with a larger scale and at higher altitude) at the edges of coronal holes. The purpose of this study was to understand the energy transport to the corona from the photosphere, where the convective motions of the magnetized fluid are believed to be the source for the coronal energy input. Although the dissipation of conventional, uniform-medium, Alfven waves is insufficient to deposit energy in the coronal plasma, we have shown that there exists a normal mode which has the form of a surface wave in this continuous medium, that has a damping rate proportional to the magnetic field gradient. In the lowest-order approximation, the dissipation is also proportional to the square of the wavenumber and independent of resistivity as long as the Lundquist number  $S$  is sufficiently large. The damping of these waves provides a potentially important mechanism

to transport energy from the photosphere to the corona where it can be effectively dissipated.

To investigate the properties of these waves in the lower atmosphere, where the magnetic Reynolds number is not as large as in the corona, we carried out the second-order calculation of their damping rates and found that it was proportional to  $S^{-0.33}$  (#29) where  $S$  is the Lundquist number. Similar results were also obtained in numerical simulations by studying the space-time evolution of the wave energy (#19) and the phase-mixing of these propagating normal modes (#24), including the effects of viscosity (#30).

The results of these studies were then applied to evaluate the energy-deposition rate in the solar atmosphere (#35). In active regions, with strong magnetic fields and mostly closed field lines, these waves suffer fast damping as they propagate upward because of their short wavelengths, as limited by the length of the field lines, and because of the small horizontal scale width of the active-region fields. Their contribution to coronal heating is furthered by the fact that they are trapped in the closed-field regions and easily dissipated within a few reflections. In the open-field regions of coronal holes, their dissipation length is much longer. Although they deposit less energy in the corona in this case, much of their energy can be absorbed into the solar-wind plasma flowing out of these magnetically open regions.

### B. 3. Anomalous Resistivity

The heating effects of anomalous resistivity and wave dissipation in coronal arches have been explored. For steady currents, exciting turbulence is not easy—electron drift velocities must exceed  $0.1 v_{te}$  to excite plasma instabilities other than drift waves. Above this threshold, the current-driven ion-cyclotron instability seems the best candidate for a resistive heating mechanism, since it is easy to excite and difficult to suppress. Detailed numerical calculations show that currents can heat arches to  $\sim 100$  eV. Convective losses then cause the instability to “percolate” at marginal stability, maintaining the temperature (#8). Ion-cyclotron waves are easier to excite than the ion-acoustic instability and can also transport heat across magnetic field lines. A current-carrying core of size  $a \sim 100$  m., set by the scale of self-field gradients in a loop, can heat a warm cocoon. Within the cocoon, temperature declines on a scale of kilometers, set by energetics. Electron Landau damping of the waves deposits energy in the

cocoon(#41). The damping electrons couple through convective cooling to the foot-point boundaries. Complications such as magnetic shear can affect these estimates, but cross-field wave transport promises a natural way to heat large volumes with small current threads.

## C. FILAMENT FORMATION

Solar filaments and prominences are believed to form as a result of runaway radiation in a magnetically insulated region (Field 1965). Both because of their intrinsic thermodynamic interest and because of their function as the most reliable flare precursor (#21, #32), we have been engaged in a long-term study of the coupled energetics and dynamics of plasmas immersed in sheared magnetic fields. Our work has emphasized parameter regimes applicable to the formation of cool and dense structures in the magnetized solar atmosphere such as prominences.

### C. 1. Thermal Instability

In order to clarify the growth and structure of *sheared-field condensations*, which were originally found numerically (Chiuderi and Van Hoven 1979), we first addressed the radiative instability in two dimensions (#28). We demonstrated how plasma dynamics can generate linear modes localized within a very narrow region of the magnetic shear layer, provided merely that the magnitude of the temperature perturbation is constrained to decay in regions where the wave vector is parallel to the equilibrium field. In practice this constraint is provided by the temperature-leveling effects of parallel thermal conduction, but the mode width is not explicitly determined by the conduction coefficient. This paper (#28) also defined the accessible regions of growth-rate/wavenumber space, and reported the results of a series of numerical calculations that characterize the mode structure of the long-wavelength, sheared-field condensations first described by Chiuderi and Van Hoven (1979).

Our next effort was devoted to an extension of the completeness of the energy- transport model applicable to thermal instability. We first introduced the effects of finite perpendicular (to  $\mathbf{B}$ ) thermal conductivity  $\kappa_{\perp}$ , which bring new unstable modes to the plasma spectrum in a way similar to the added dynamic effects of resistivity. In an analytic study (#36) which complemented a parallel computational effort, we detailed the dynamics and structure of



these  $\kappa_{\perp}$  modes, and obtained their growth rates for arbitrary electron-ion thermal coupling (#31). A crucial result is that these modes grow preferentially at an intermediate position in the magnetic shear layer where the parallel sound-speed (mass-flow) rate is dominant (#53). Thus the radiative cooling occurs under isobaric conditions; that is, with a large accompanying density enhancement which drives the cooling at a nonlinearly accelerating rate (see, e.g., #47).

Any dynamic phenomenon which is observable on the sun must represent a large-amplitude state, so that one must provide a fully nonlinear treatment. We have attacked the filament-formation problem from this perspective, which required the development of a new 2.5-dimensional simulation code. Our first results described the accelerating cooling of the long-wavelength radiative modes, which eventually drive the local temperature down at  $10^3 \times$  the initial rate (#34).

During this period we discovered the crucial dynamic effects of the short-wavelength  $\kappa_{\perp}$  modes described above. The fact that the sound-speed rate is the fastest one in the problem means that a significant parallel (to  $\mathbf{B}$ ) mass flow occurs. To demonstrate this effect, we then performed a comprehensive set of supercomputer experiments (#47, #53) with various excitations of these modes, including the empirically relevant case of a noise-spectrum input.

We were able to show, in this series of simulations, that an even more dynamic effect arises nonlinearly (#47). After an early phase of isobaric cooling, the local pressure begins to drop, leading to a strong, parallel, mass inflow to the condensation. The final state produced in these simulations (resulting from a variety of different initial states) provides a very good match to the observed properties of a coronal prominence/filament. The density increases by  $10^2 \times$  from coronal values, while the temperature drops to  $10^{-2} \times$  the initial value. In addition, in a new and unique result, the angle between the filament axis and the local magnetic field agrees with observations (#47). These results indicate that we now know how filaments form, dynamically and energetically, in the sheared field of the ambient solar corona.

## C. 2. Prominence Condensation

We have proposed and demonstrated a new mechanism that triggers the formation of a coronal prominence in an otherwise thermally stable magnetic arcade. Based on the conception that heat in the corona is supplied from the photosphere through various mechanisms, such as Alfvén waves, and is guided by the magnetic field, we argue that, if the energy supply to the apex is curtailed, thermal conduction may not be able to sustain the apex temperature, resulting in the initiation of local cooling. By numerical simulation, we have demonstrated that a prominence can indeed form naturally in this model (#51) which exhibits strong siphon flow from the chromosphere as needed to obtain sufficient mass in the condensation. This multi-phase process results from a single causative input, in contrast with other previous models which require multi-step events in the corona (Poland and Mariska 1986). We further simulated the effect of the bending of the magnetic field due to the heavy condensed materials at the apex, where the field lines are essentially horizontal before the condensation, by modifying the gravity projected along the field. A magnetic-field "hammock" is formed, as a result of the modified geometry, which stably traps the condensed plasma in the form of a levitated apex prominence of the Kippenhahn-Schlüter (1957) type.

## D. FLARE ENERGY RELEASE

The energy which drives a solar flare is believed to be stored in the local, highly stressed, active-region, magnetic field. In order to release this energy, one must change the topology of (i.e., "reconnect") this field, using a resistive process. The only known truly dynamic mechanism to accomplish this is by an instability called the *tearing mode* which has been a long-time focus of this program.

### D. 1. Magnetic Reconnection

Among our early efforts in evaluating the application of resistive tearing to the flare-release mechanism was a quantitative study of the high-conductivity (high Lundquist number  $S$ ) growth and scaling of this instability (#5).

We followed this linear study with a series of *nonlinear* simulations of the *energy output* of

the resistive tearing instability of a plane field-shear layer. These computations, which held the high- $S$  record (at  $10^6$ ) for a number of years, showed that more than 30% of the stored magnetic energy could be liberated (#15, #18, #26). Comparable results were obtained in a different set of simulations performed with a cylindrical-pinch (coronal loop) field (#20).

A second nonlinear focus of our flare/reconnection studies involves the possible speed-up effects of MHD *turbulence*, whether through modifications of the resistivity or of the dynamics. Our first effort diagnosed the temporal evolution of a (time-averaged) anomalous resistivity proposed by Biskamp and Welter (1983). We demonstrated (#38) that the sign and magnitude of the "resistivity", claimed to depend on the turbulent average values of (kinetic energy)–(magnetic energy), quickly oscillated and decayed without a significant effect on the tearing mode.

An alternative proposed mechanism for the modification of magnetic reconnection involves the effect of co-existing MHD fluctuations. Matthaeus and Lamkin (1986) have claimed that turbulence can drive reconnection, and we have completed an investigation of this question. Our method was to apply varying levels of wide-spectrum noise input to a tearing-mode-unstable magnetic field, similar to that used by Matthaeus and Lamkin, but to follow the temporal evolution of the system for much longer times. Using either our periodic or our ADI 2.5-D codes (see Appendix II), which have both been previously used to study nonlinear tearing, we can compute for periods commensurate with the saturation time. We find that, when starting with low levels of noise, the expected fastest-growing tearing-mode excitation appears, and grows to its usual nonlinear saturation or stored-energy level. As the turbulence amplitude is progressively increased to the natural stored magnetic-energy level, near that chosen by Matthaeus and Lamkin as an initial excitation, the tearing mode is obscured and modified (buffeted about, actually), but the degree of energy release is unchanged. The results of these simulations are reported in Deeds and Van Hoven (#43). These two simulations formed the core of the Ph. D. dissertation of D. Deeds (#45).

## D. 2. Radiative Effects

In parallel with these nonlinear studies, we began an investigation of the effects on resistive tearing of the addition of those energy-transport contributions which are important in

the solar application; namely, ohmic heating, thermal conduction and (predominantly) radiation loss (#16). In particular, this effort produced a very complete (and clear) treatment of this complex of effects in an analytic boundary-layer calculation (#10).

We then expanded this study, motivated by the close spatial and temporal relation between filaments and flares, to a series of studies on the interaction between radiative and reconnection instabilities (#7). Computational investigations of this coupling, which arises from the temperature dependence of collisional resistivity, produced a number of publications detailing the growth rates and mode structures (#7, #11, #14, #25), and the additional effects of compressibility and viscosity (#22). These studies also resulted in the completion of a Ph.D. dissertation for T. Tachi (#12).

The crowning result of this series of energy-transport modelling efforts was the discovery and elucidation of the *radiative-tearing instability*, a very fast magnetic-reconnection mode (#13, #17). This instability's high speed arises from the effect of radiative cooling on the Coulomb resistivity ( $\eta \propto T^{-\frac{3}{2}}$ ) which produces a very effective resistive-diffusion reconnection layer.

### D. 3. Observable Diagnostics

As a result of the physical complexity of the solar flare event, we have early-on sought to characterize and learn from the relative simplicity of the preflare state. To this end the PI served as a group leader in the COSPAR sponsored Flare Build-up Study, devoted to precursors and onset (#21, #32), with particular attention to the connection (both spatial/magnetic and temporal) with pre-existing filaments. A second diagnostic effort sought to evaluate the prospects for flare x-ray polarimetry to distinguish between thermal and non-thermal models of the impulsive phase (#33, #39).

We have previously investigated a loop-flare model based on a resistive-pinch configuration with a force-free magnetic field (Mok and Van Hoven 1982). Under the line-tied boundary conditions presented to the loop field by the dense photosphere, the fundamental unstable mode is localized at the apex of the loop where the local magnetic field is nearly azimuthal. The induced electric (DC) field, as a result of magnetic reconnection, can accelerate charged particles in a very efficient manner along the local, closed, field lines

like a synchrotron (#9). This picture is consistent with observations indicating a compact, partially polarized, microwave source at the loop apex during a flare.

All reconnection flare models naturally lead to thick-target x radiation as the energetic electrons drift across the field lines and are directed down toward the photosphere. Since electron beams going through a plasma are often unstable and thus subject to scattering by a variety of plasma oscillations, their transmission through the solar atmosphere is an important question. Based on their energy spectrum, deduced from hard x rays, we have studied a potentially unstable plasma wave (#23) driven by these fast electrons through the anomalous Doppler resonance ( $\omega - \mathbf{k} \cdot \mathbf{v} - \Omega_e = 0$ ). The instability condition was shown to be  $\omega_{pe}/\Omega_e \geq 1.9$  for a power-law distribution with a wide range of hardnesses, where  $\omega_{pe}$  is the local plasma frequency and  $\Omega_e$  is the electron gyrofrequency. This instability threshold is often crossed when the fast electrons pass the transition region where the local density and plasma frequency rise. The slowing down of the beam, as a result of pitch-angle scattering, can cause intense heating at a location different from the one predicted by conventional models (#23). There are also a few unexplained aspects of the UV observations, which may be attributed to this instability.

Another flare-diagnostic study (#55) explored a model for impulsive energization of the arch foot points by bombarding them with slugs of magnetically-driven plasma. These plasmoids are released when magnetic field lines reconnect at the arch peak. The plasma in the resulting magnetic pockets then accelerate as the field lines retract, slamming plasma into the dense footpoint layers and heating them rapidly. This can account for impulsive x-ray emission.

## E. SPECIAL TOPICS

An early effort in this solar plasma theory program was devoted to the evaluation of the influence on radiative opacity of plasma shielding in the solar interior. Some effects which could reduce the central temperature of the sun, and thereby lessen the neutrino-flux discrepancy, were described in the Ph.D. dissertation of R. O. Hunter (#2).

At the other extreme of the heliosphere, we have performed two studies of plasma flow

past Jupiter's satellite Io. In the first, we verified that 3-D MHD simulations could exhibit the formation of "Alfven wings", a region of standing Alfven-wave perturbations generated by the interaction of the Jovian magnetic field with Io's ionosphere. We also discovered that standing slow-mode perturbations were present, a feature that had not been quantitatively modeled previously (#37).

In a second study, we examined the effects that mass pickup can have on the plasma temperature in an MHD approximation. Although previous studies have claimed that pickup can heat the torus plasma, we showed that such thermal pickup effects are controlled by the local sonic Mach number. The process is nonlinear because pickup can slow the flow appreciably. Thus, less heating than anticipated, or perhaps even cooling, may occur locally near Io as a result of ionization (#46).

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## APPENDIX I: List of UCI STT Publications

1. The Stability of a Diffuse Linear Pinch with Axial Boundaries, *Phys. Fluids* **24**, 1092 (1981); G. Einaudi and G. Van Hoven.
2. Plasma Physics Aspects of the Solar Neutrino Problem, Ph.D. Dissertation (1981); R.O. Hunter, Jr.
3. A Boundary-Coupled Generalization of the Newcomb Stability Criterion, *Phys. Fluids* **25**, 1355 (1982); A. Ray and Van Hoven.
4. The Structure, Stability and Flaring of Solar Coronal Loops, *Memoirs Ital. Astron. Soc.* **53**, 441 (1982); Van Hoven.
5. The Growth of the Tearing Mode: Boundary and Scaling Effects, *Phys. Fluids* **26**, 117 (1983); R.S. Steinolfson and Van Hoven.
6. Closed and Open Magnetic Fields in Stellar Winds, *Astrophys. J.* **266**, 823 (1983); Steinolfson and D. J. Mullan.
7. Energy Dynamics in Stressed Magnetic Fields: The Filamentation and Flare Instabilities, *Astrophys. J.* **268**, 860 (1983); Van Hoven, Steinolfson, and T. Tachi.
8. Turbulent Resistive Heating of Solar Coronal Arches, *Astrophys. J.* **269**, 690 (1983), G. Benford.
9. Microwave Signatures from a Reconnecting Plasma Pinch with Application to Loop Flares, *Astrophys. J.* **275**, 901 (1983); Y. Mok.
10. Energetics and The Resistive Tearing Mode: Effect of Joule Heating and Radiation, *Physics of Fluids* **26**, 2590 (1983); Steinolfson.
11. The Effects of Ohmic Heating and Stable Radiation on Magnetic Tearing, *Physics of Fluids* **26**, 2976 (1983); Tachi, Steinolfson and Van Hoven.
12. Radiative and Reconnection Instabilities: Filaments and Flares, Ph.D. Dissertation (1984); T. Tachi.



13. Radiative Tearing: Magnetic Reconnection on a Fast Thermal Instability Time Scale, *Astrophysical Journal* **276**, 391 (1984); Steinolfson and Van Hoven.
14. Radiative and Reconnection Instabilities: Filaments and Flares, *Astrophys. J.* **280**, 391 (1984); Van Hoven, Tachi and Steinolfson.
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21. Flare Precursors and Onset, *Advances in Space Research*, Pergamon (1984), Vol. 4, No. 7, pp. 95-103; Van Hoven and G. J. Hurford.
22. Radiative and Reconnection Instabilities: Compressible and Viscous Effects, *Solar Physics* **95**, 119 (1985); Tachi, Steinolfson and Van Hoven.
23. Propagation of Energetic Electron Streams in Solar Flares, *Solar Physics* **95**, 181 (1985); Mok.
24. Resistive Wave Dissipation on Magnetic Inhomogeneities: Normal Modes and Phase Mixing, *Astrophysical Journal* **295**, 213 (1985); Steinolfson.

25. A Unified Treatment of the Filament and Flare Instabilities, in IAU Symposium 107, *Unstable Current Systems and Plasma Instabilities in Astrophysics* (eds.: M. R. Kundu and G. D. Holman), Reidel, pp. 263-71 (1985); Van Hoven.
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## APPENDIX II: UCI STT Numerical/Simulation Codes

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### A.II.A. Inverse-Power MHD Linear-Instability Code

Computational studies of the behavior of ideal and resistive plasma instabilities in a sheared magnetic field often encounter a problem of conflicting length scales; in particular, the characteristic width of an unstable perturbation is often much smaller than the characteristic length (e.g., the shear scale  $a$ ), of the equilibrium. One strategy is to retain second-order accuracy in the finite-difference expressions for first- and second-order derivatives by employing an appropriate coordinate transformation (e.g., Van Hoven and Mok 1984). We make the change of variable  $r = \sinh^{-1}[b(y - y_c)/a]$ , where  $y_c$  marks the region of high resolution and  $b$  is an adjustable parameter which controls the magnitude of the resolution enhancement. Away from the region  $y \approx y_c$ ,  $r$  effectively represents a logarithmic scale. This transformation is more flexible than the one used by Van Hoven and Mok (1984) since it remains well behaved for  $y < y_c$ .

The full linearized system of MHD equations may be expressed as

$$\partial \mathbf{X} / \partial t + \mathcal{L} \cdot \mathbf{X} = 0 \quad (\text{A1})$$

where  $\mathbf{X}$  is a vector whose elements are the perturbation variables and  $\mathcal{L}$  is the corresponding matrix differential operator (in the transformed variable  $r$ ). Working on an  $N$ -point spatial grid and using three-point spatial difference formulae, we write equation (A1) in finite-difference form at each grid point ( $i = 1, 2, \dots, N$ ), implementing boundary conditions at the first and last grid points. To obtain exponentially growing solutions, we use an inverse-power method (Conte and de Boor 1980, p. 193); the growth rate of a desired solution is treated as an eigenvalue and estimated to be  $\nu_0$ . Using a block tridiagonal algorithm (e.g., Roache 1976, pp. 345-349) to solve the set  $N \times 7$  simultaneous MHD equations, iteration is then performed according to the relation

$$\mathbf{X}^{N+1} = (\mathcal{L} + \nu_0 \mathcal{I})^{-1} \mathbf{X}^N$$

where  $\mathcal{I}$  is the identity matrix. The vector  $\mathbf{X}^N$  is found in general to converge rapidly to the eigenvector  $\mathbf{X}$ , with corresponding eigenvalue  $\nu$ , if  $\nu_0$  is initially chosen to be sufficiently close to  $\nu$ .

### A.II.B. Nonlinear 1-D Thermal and Gravitational Hydrodynamics Code

It is well known that the solar coronal plasma is locally unstable to rapid cooling and condensation due to energy losses from optically thin radiation. Throughout most of the corona this effect is mitigated by thermal conduction; thus the corona is mostly hot and tenuous. However, cool, dense plasma is observed to form in highly localized regions associated with sheared magnetic field configurations. An understanding of the thermal stability of plasmas under such conditions is complicated by the complex, three-dimensional, magnetic geometry, and by the thermal connection to cooler, denser regions by means of magnetic field lines that thread both the corona and the chromosphere. Theoretical progress can be made by considering several related model problems with simpler geometry and boundary conditions. For several years we have studied radiative instabilities in two-dimensional sheared magnetic geometry. These studies have shed light on the plasma dynamics transverse to the magnetic field, and on the role played by the sheared field in thermally insulating the cool plasma from the surrounding hot corona. Such models say nothing about the thermal connection to the cool and dense chromosphere provided by dynamics parallel to the magnetic field.

Recently we have developed a fully nonlinear, one-dimensional, gravitational hydrodynamics code to study these Thermal-connection effects. As this model considers only parallel flows, magnetic field evolution is ignored, as are curvature effects. The ends of the field line are considered anchored in the chromosphere, where a variety of boundary conditions (e.g., thermal insulation, inflow/outflow, constant temperature, exponential density, etc.) can be applied. In addition to the usual adiabatic terms, the energy equation includes thermal conduction ( $\kappa \sim T^{\frac{5}{2}}$ ), optically thin radiation, and variable coronal heating. The flows respond to pressure and (parallel) gravity forces. A simple artificial viscosity (proportional to the minimum mesh spacing) is used to prevent nonlinear

numerical instability at small wavelengths. All physical terms are advanced by using a leap-frog algorithm on a non-uniform Eulerian grid. The equation of motion is modified with semi-implicit terms (proportional to the timestep, Harned and Schnack 1986) which allow for numerically stable computations to be performed on time scales dictated by interesting physical processes, rather than by the usual CFL condition.

### **A.II.C. Nonlinear 2.5-D ADI Incompressible MHD Magnetic- Reconnection Code**

A pervasive phenomenon in solar and space physics is magnetic reconnection: the spontaneous annihilation of magnetic field and its corresponding conversion into plasma kinetic and internal energy. These processes are of interest because they occur on timescales much shorter than that indicated by simple resistive dissipation, yet much longer than the timescale dictated by Alfvén or sound waves. Numerical studies of the nonlinear evolution of reconnection events thus require that special computational methods be used. Over the past few years we have developed a 2.5-dimensional resistive MHD code suitable for these studies. This code employs a coordinate transformation (App. II. A:) in the direction transverse to the primary magnetic field to allow for the enhanced spatial resolution required by astrophysical values of the Lundquist number. The assumption of incompressibility eliminates sonic and magnetosonic waves, while retaining the essential reconnection physics (Furth *et al.* 1963, Van Hoven and Cross 1973). Joule heating, thermal conduction, and optically thin radiation are also included in the model. Potentially unstable spatial oscillations at the mesh size, arising from quadratic nonlinearities in the MHD equations, are controlled by a simple spatial filter (Schnack, Baxter, and Caramana 1984). The time advance uses the Alternating-Direction-Implicit (ADI) method (Schnack and Killeen 1980) to compute on the long time scales defined by the reconnection process. Time centering is accomplished by Newton iteration of the nonlinear terms (Finan and Killeen 1981). In this way the time step is determined by the requirements of accuracy and efficiency rather than by that of numerical stability of the fastest waves.

#### **A.II.D. Nonlinear 2.5-D Compressible and Resistive MHD Thermal-Instability Code**

The structure and operation of this code are described at some length in the Appendix of Sparks, Van Hoven and Schnack (#47).

#### **A.II.E. Nonlinear 2.5-D Resistive MHD Spectral Code**

This code was used for the *first* nonlinear simulation of the resistive tearing mode (Van Hoven and Cross 1973) and is described in that publication. It has since been partially vectorized and a predictor-corrector time advance installed (#38).

#### **A.II.F. Modified 3-D MHD Code in Spherical Geometry**

Our time-dependent, three-dimensional MHD code solves the nonlinear, compressible MHD equations in spherical coordinates using a two-step Lax-Wendroff method. This algorithm is useful for problems where advection of flow is important, including the capturing of shocks and discontinuities (Richtmyer and Morton 1967). The code was originally developed to study three-dimensional plasma flow past Io (#37 gives a brief description), but has been adapted for solar problems. We have considered 2-D problems with azimuthal symmetry (#48) and full 3-D applications (#58). Our code can be used either for problems where boundaries are imposed at arbitrary values of  $r$ ,  $\theta$ , and  $\phi$ , or for global problems where the only boundaries occur at the solar surface and an outer radial location (solution at the poles of the spherical coordinate system is then accomplished by differencing a limit form of the MHD equations, similar to a form of the gas-dynamic equations derived by Bohachevsky and Mates 1966). For future problems, we plan to add semi-implicit terms to the code (Harned and Schnack 1986) to allow us to also follow the slow build-up of energy in the magnetic field while using large time steps. These terms are automatically turned off during the dynamic portions of the computation.

#### **A.II.G. Modified 3-D MHD Code in Cartesian Geometry**

The structure and operation of this code, which was used in #40, are described in the Appendix of Mikić *et al.* (1988).



### **A.II.H. 3-D MHD Code in Cylindrical Geometry**

We have recently developed a 3-D cylindrical MHD code which is similar to the Cartesian code described by Mikić, Barnes, and Schnack (1988). It has a Fourier representation in the azimuthal coordinate, with finite differences in the  $r$  and  $z$  coordinates. It has been applied to the dynamics and stability of coronal loops (#52).

### **APPENDIX III: Abstracts of UCI STT Publications**

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(see following pages)

# Stability of a diffuse linear pinch with axial boundaries

Giorgio Einaudi

*Istituto di Fisica, Università di Pisa, Pisa, Italy*

Gerard Van Hoven

*Department of Physics, University of California, Irvine, California 92717*

(Received 11 August 1980; accepted 27 February 1981)

A formulation of the stability behavior of a finite-length pinch is presented. A general initial perturbation is expressed as a uniformly convergent sum over a complete discrete  $k$  set. A variational calculation is then performed, based on the energy principle, in which the end-boundary conditions appear as constraints. The requisite Lagrange multipliers mutually couple the elemental periodic excitations. The resulting extended form of  $\delta W$  still admits a proper second-variation treatment so that the minimization and stability considerations of Newcomb remain applicable. Comparison theorems are discussed as is the relevance of this end-effect model to the stability of solar coronal loops.

## I. INTRODUCTION

It is well known<sup>1</sup> that the study of the hydromagnetic stability of an equilibrium configuration can be reduced to the solution of the linearized equation of motion

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = F(\xi), \quad (1)$$

where  $\xi$  is the Lagrangian displacement of a fluid element from its equilibrium position. The force operator  $F$  can be written in the form

$$F(\xi) = \gamma \nabla \rho_0 (\nabla \cdot \xi) + \nabla (\xi \cdot \nabla) \rho_0 + \frac{1}{4\pi} [(\nabla \times Q) \times B_0 + (\nabla \times B_0) \times Q],$$

where  $\rho_0$ ,  $p_0$ , and  $B_0$ , respectively, are the equilibrium density, pressure, and magnetic field,  $\gamma$  is the ratio of specific heats and  $Q = \nabla \times (\xi \times B_0)$  is the first-order perturbation in the magnetic field. By solving (1) with the proper boundary and initial conditions, it is possible, in principle, to follow in time any small motion about an equilibrium state. However, one can determine if a system is stable or not in an easier way by using an energy principle,<sup>1,2</sup> if  $F$  is a self-adjoint operator. To do so, one uses the variation in kinetic energy  $\delta K$  (a volume integral of  $\frac{1}{2} \rho_0 |\dot{\xi}|^2$ ) and the change in the potential energy, which is a quadratic form

$$\delta W = -\frac{1}{2} \int_V d\tau \xi \cdot F(\xi) \quad (2)$$

not involving  $\dot{\xi}$  because of the form of (1). From (1) and (2) it follows that

$$\begin{aligned} \delta \dot{W} + \delta \dot{K} &= \frac{1}{2} \int_V d\tau [\dot{\xi} \cdot F(\xi) - \xi \cdot F(\dot{\xi})] \\ &= \delta W_s(\dot{\xi}, \xi) - \delta W_s(\xi, \dot{\xi}), \end{aligned}$$

where

$$\begin{aligned} \delta W_s(\xi, \xi') &= \frac{1}{2} \int_S d\sigma \hat{n} \cdot \{ \xi \cdot [(\xi' \cdot \nabla) \rho_0 + \gamma \nabla \cdot \xi' - B \cdot Q'] \\ &\quad + B(\xi \cdot Q') \} \end{aligned}$$

is the surface contribution to  $\delta W$  when  $\xi = \xi'$ ,  $S$  is the

boundary of the plasma, and  $\hat{n}$  is the outward unit normal to the surface. It is clear that the requirement for the operator  $F$  to be self-adjoint depends upon the boundary values  $\xi$  has to satisfy. The perturbations we have chosen provide this condition in the form  $\delta W_s = 0$ .

Newcomb<sup>3</sup> investigated the stability of a cylindrically symmetric and infinitely long configuration of plasma and magnetic field (a diffuse pinch) bounded on the outside by a perfectly conducting wall. In this case  $\delta W_s$  is obviously zero. We want to investigate the stability of a similar structure, except that we introduce an inhomogeneity along the axis; that is, we consider the equilibrium density to have the form

$$\rho_0(z) = \begin{cases} \rho_1, & |z| \leq L \text{ (I)}, \\ \rho_2 \gg \rho_1, & |z| > L \text{ (II)}, \end{cases} \quad (3)$$

where  $2L$  is the length of the region (I) of interest. Since the Alfvén speed  $v_A$  is inversely proportional to  $\rho_0^{1/2}$ , it is much larger in region I than in region II; it follows that, on a short time scale, we can consider the perturbation  $\xi$  to be zero in regions II, and thus require it to satisfy specific boundary conditions at  $z = \pm L$ . In order to solve the problem using an energy principle these conditions have to be such that  $\delta W_s$  is zero at the "ends". A possible choice is

$$\xi_\perp(-L) = \xi_\perp(L) = 0, \quad (4a)$$

$$\xi_\parallel(-L) = \xi_\parallel(L), \quad (4b)$$

$$\frac{d}{dz} \xi_\perp(-L) = \frac{d}{dz} \xi_\perp(L), \quad (4c)$$

$\xi_\perp$  and  $\xi_\parallel$  being, respectively, the components of  $\xi$  perpendicular and parallel to  $B_0$ . Notice that, since conditions (4) have to hold at every time, the boundary conditions on  $\xi$  are the same. It is easy to see the physical meaning of conditions (4) by considering the perturbation in the magnetic field  $Q$ . Because of (4a),  $Q_\parallel$  vanishes at  $z = \pm L$  and, therefore, the normal component of the magnetic field across the boundary between fluids I and II is continuous even if  $Q$  vanishes in fluid II. It follows that condition (4a) is the necessary and sufficient condition to model the inertial anchoring of the lines of force of the magnetic field at the surfaces  $z = \pm L$ . This an-

# A boundary-coupled generalization of the Newcomb stability criterion

Alak Ray<sup>a</sup> and Gerard Van Hoven

Department of Physics, University of California, Irvine, California 92717

(Received 12 August 1981; accepted 5 May 1982)

In magnetohydrodynamic problems in which there are boundaries in the axial or azimuthal symmetry directions, the energy principle involves multiple nonorthogonal trial perturbations. The subsequent Euler-Lagrange coupling of these allowed excitations necessitates a generalization of the Newcomb necessary-and-sufficient stability criterion. This extension, in which the absence of a conjugate point (instead of a simple zero) provides stability, is described in this paper. An important aspect of the proof involves the treatment of certain asymmetries which arise in the presence of the anisotropy caused by the magnetic field. The general method described here has applications to finite-length axial laboratory pinches and to astrophysical plasmas with rooted magnetic fields.

## I. INTRODUCTION

The Newcomb criterion<sup>1</sup> provides a straightforward test, based on the Energy Principle,<sup>2</sup> for determining the stability of a periodic, or infinitely long, cylindrical pinch. Significant simplification is obtained, in Newcomb's theory, by expressing the variational trial functions in terms of the *orthogonal* complete set

$$\xi_k(r) = \xi(r; m, k) \exp[i(m\theta + kz)], \quad (1)$$

which fits the pinch symmetry. One then determines ideal magnetohydrodynamic stability by the absence of zeros of the properly initialized Euler-Lagrange solution  $\xi_k(r; m, k)$  for all  $m$  and  $k$ , a test which can be performed by a straightforward numerical procedure.<sup>3</sup>

If, however, such a linear pinch has a limited extent in either symmetry direction, with specified boundary conditions, then at least two test functions of the form of (1) are required, and they become coupled in the minimization procedure.<sup>4</sup> Situations of this kind, which provide improved stability<sup>5</sup> by restricting the allowed perturbations, arise for finite-length line-tied pinches<sup>6</sup> and for arcade-like half pinches<sup>7</sup> each of which models structures in the solar corona.

In this paper, we generalize the primary Newcomb criterion<sup>1</sup> to the case of a bounded, magnetized plasma. The boundary conditions both quantize the wavenumber  $k$  and destroy the energy-integral orthogonality of the members of the allowed trial-perturbation set, including that given in (1) and other alternative complete sets.<sup>6</sup> This situation then leads to coupled Euler-Lagrange equations, whose solutions provide a test for stability. Such problems are known in the Calculus of Variations (e.g., Gelfand and Fomin<sup>8</sup>), but the available criteria do not apply to the situation in which helical magnetic fields thread the plasma. In this case, the resultant anisotropy destroys the  $(k, k') \rightarrow (-k, -k')$  symmetry of certain coupling coefficients, which is taken as an essential assumption of previous proofs.

Section II briefly describes the energy-principle formulation required for bounded plasmas. Section III provides the statement and a proof of the necessary generalization of the Newcomb criterion. Finally, Sec. IV discusses applications and conclusions.

## II. ENERGY PRINCIPLE FORMULATION

In this section we will follow the treatments of Bernstein *et al.*<sup>2</sup> and Newcomb,<sup>1</sup> substantially using the notation of the latter. The energy principle states that a plasma is stable if the (potential) energy integral

$$\delta^2 W = -\frac{1}{2} \int_V d^3r \xi \cdot F(\xi) \quad (2)$$

is positive, where  $F$  is a self-adjoint operator representing the ideal magnetohydrodynamic force<sup>2</sup>, and is a function of equilibrium quantities. Newcomb applied this method to the linear pinch and, after expressing the displacement  $\xi(r)$  in an orthogonal set as in (1), he minimized  $\delta^2 W$  with respect to  $\xi_\theta$  and  $\xi_z$ . The resulting form

$$\delta_k^2 W = \frac{\pi}{2} \int_a^b dr [f(\xi_k')^2 + g\xi_k'^2], \quad (3)$$

given in terms of  $\xi_k(r) = \xi(r; m, k)$  [where  $\xi_k(a) = 0 = \xi_k(b)$ ] and  $\xi_k' = d\xi_k/dr$ , is minimized by the solutions of the Euler-Lagrange equation

$$(f\xi_k')' - g\xi_k = 0 \quad (4)$$

for each value of  $m$  (suppressed) and  $k$ . Newcomb then showed,<sup>1</sup> in a very careful way, that

$$\delta_k^2 W > 0 \Leftrightarrow \xi_k(r) \neq 0 \quad (5)$$

for  $0 < a < r < b$  (or in certain independent subintervals thereof), with  $\xi_k(r)$  having proper initial values at  $r = a$ . This allows the determination of stability to be made from a simple numerical integration<sup>3</sup> of (4).

When a finite plasma is considered, however, with specified axial or azimuthal boundary conditions, the Newcomb formulation can no longer be used. One way of seeing this is that the functions  $f$  and  $g$  are not even in  $k$  and  $m$ . Thus, a resonant "standing wave" solution [ $\xi_k = \xi_{-k}$  in the notation of (1)], which will satisfy end conditions on a finite-length pinch, for example, cannot be constructed from (4). There are also phase relations<sup>1</sup> among the components of  $\xi_k$  which prevent their vanishing simultaneously on specified surfaces.

To circumvent this problem, one must either *constrain* the variation of (2) as described by Einaudi and Van Hoven<sup>4</sup> or devise special forms of the minimizing test functions which have the boundary values built into them.<sup>9,10</sup>

<sup>a</sup> Present address: Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India.

## THE STRUCTURE, STABILITY AND FLARING OF SOLAR CORONAL LOOPS

Gerard VAN HOVEN  
*Department of Physics*  
*University of California*  
*Irvine, Calif. 92717, USA*

### Abstract

A review is given of recent progress in the theory of the magnetohydrodynamic behavior of coronal loops, beginning with a brief characterization of the observations. The equilibrium magnetic field is described, along with the consequences of the empirical requirement for short-term, or infinite-conductivity, stability which is shown to be dominated by the end-effect influence of the quasi-rigid photosphere. A new loop-flare model is then developed, which takes account of the finite loop length. The primary resistive-sausage-mode instability exhibits the necessary threshold behavior, and produces a number of spatially and energetically distinct flare-release manifestations.

# The growth of the tearing mode: Boundary and scaling effects

R. S. Steinolfson

*Department of Physics, University of California, Irvine, California 92717*

G. Van Hoven<sup>a)</sup>

*Osservatorio Astrofisico di Arcetri, Firenze, Italy*

(Received 29 April 1982; accepted 28 September 1982)

The linear development of the resistive tearing instability in a sheet pinch is investigated numerically. Particular emphasis is placed on effects which differentiate magnetic tearing in astrophysical situations from that in laboratory devices. These include extreme values of the parameters determining the mode growth and a variety of boundary conditions. Eigenfunction profiles for long and short wavelengths are computed and the applicability of the "constant  $\Psi$ " approximation is investigated. Nearby conducting walls tend to validate this condition and reduce the growth rate, especially for the long wavelength modes which, otherwise, disturb a larger region of the plasma than do short wavelength modes. Finally, the growth rate  $p$  is computed for values of the magnetic Reynolds number  $S$  up to  $10^{12}$  and of the dimensionless wavelength parameter  $\alpha$  down to  $10^{-3}$ . The results demonstrate, without approximation, the  $S^{2/5}$  scaling of  $p$  at large  $\alpha$  (constant  $\Psi$ ) and the  $S^{2/3}$  scaling at small  $\alpha$  (nonconstant- $\Psi$ ). The  $\alpha$  and  $S$  variation of the growth maximum, which would provide the dominant excitation in the absence of nearby boundaries, is specified for both single- and multiple-tearing layers. The growth maximum is shown to occur in a parametric regime where the constant- $\Psi$  approximation is *not* valid.

## I. INTRODUCTION

The study of magnetic reconnection has a long history, dating from Dungey's proposal<sup>1</sup> of such a process to explain the energy release of solar flares and other astrophysical explosions. Two primary reconnection mechanisms have been developed; one a true instability<sup>2</sup> and the second a static, externally driven, resistive diffusion.<sup>3</sup>

The former mechanism, resistive magnetic tearing, has been treated most often with the original analytic boundary-layer theory of Furth, Killeen, and Rosenbluth<sup>2</sup> (cited hereafter as FKR) and has been used most extensively for laboratory-fusion applications, often in cylindrical geometry.<sup>4</sup> Many of these calculations depend upon certain ordering assumptions<sup>5</sup> or on a long-wavelength approximation.<sup>4,6</sup>

Previous investigators using numerical computations<sup>7-10</sup> have not particularly investigated the adequacy of these analytic simplifications, nor have they covered a wide range of parameter space. Many recent workers<sup>11,12</sup> have concentrated on the study of nonlinear effects<sup>13,14</sup> over a practically limited span of physical conditions.

We have several near- and far-term motivations in the present work. We wish to assemble a complete and self-consistent numerical model of resistive magnetic tearing in order to (a) verify and relate the results of the principal approximations used in analytic analyses, and (b) to investigate the solutions and their growth-rate scalings<sup>2,4,6</sup> over a large range of the primary parameters which include parametric values applicable to the solar atmosphere.<sup>15</sup> The computations described here cover the linear behavior for a variety of boundary conditions, as a first step to a series of numerical calculations probing the nonlinear evolution and eventual

saturation behavior of linearly unstable modes.<sup>13,14</sup> We are also, ultimately, interested in astrophysical and energy-release applications of the tearing mode,<sup>15</sup> for which the magnetic Reynolds numbers and wavelengths are larger than, and the boundary conditions and relevant diagnostics different from, those used for laboratory purposes.

In the present paper we employ a model which uses several of the same assumptions as FKR.<sup>2</sup> Our approach to obtaining a solution to the dynamic equations differs from that used by FKR, however, in that we solve the time-dependent equations numerically throughout the entire region and do not follow a boundary-layer approach in which solutions applicable to two different regions must be matched at an appropriate interface. We, therefore, do not require either a "constant- $\Psi$ "<sup>2</sup> or a long-wavelength approximation,<sup>4</sup> although we examine solutions covering the range from where constant  $\Psi$  would be expected to be valid (relatively short wavelengths) to where it should not be valid (long wavelengths). By computing simulations over a wide span in  $S$  ( $10^2 < S < 10^{12}$ ) and  $\alpha$  ( $10^{-3} < \alpha < 1$ , where  $\alpha$  is inversely proportional to the wavelength) we are able to verify various scaling laws for the growth rate which have appeared in the literature,<sup>2,4,6</sup> without benefit of a boundary-layer approach and any related subsequent approximations.

## II. THE RESISTIVE EQUATIONS AND THEIR NUMERICAL SOLUTION

The basic equations<sup>2</sup> and the numerical procedure<sup>16</sup> used to solve them are well-established, so they will just be briefly reviewed here. We assume that collisional magnetohydrodynamic (MHD) theory<sup>17</sup> is applicable, that the plasma is incompressible and inviscid with constant isotropic resistivity  $\eta$ , and that gravitational effects are negligible. The necessary equations can then be written in cgs units as fol-

<sup>a)</sup>Permanent address: Department of Physics, University of California, Irvine, California 92717.

## CLOSED AND OPEN MAGNETIC FIELDS IN STELLAR WINDS

D. J. MULLAN

Bartol Research Foundation of The Franklin Institute

AND

R. S. STEINOLFSON

Physics Department, University of California, Irvine

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### ABSTRACT

We report on a numerical study of the interaction between a thermal wind and a global dipole field in the Sun and in a giant star. In order for closed field lines to persist near the equator (where a helmet-streamer-like configuration appears), the coronal temperature must be less than a critical value  $T_c$ , which scales as  $M/R$ . This condition is found to be equivalent to the following: for a static helmet streamer to persist, the sonic point above the helmet must not approach closer to the star than 2.2–2.6 stellar radii. Implications for rapid mass loss and X-ray emission from cool giants are pointed out. Our results strengthen the case for identifying empirical dividing lines in the H-R diagram with a magnetic topology transition locus (MTTL). Support for the MTTL concept is also provided by considerations of the breakdown of magnetostatic equilibrium.

*Subject headings:* hydromagnetics — stars:coronae — stars:winds

### 1. INTRODUCTION

The main question which we wish to examine here is the following: What condition must be satisfied in order that closed magnetic field lines can persist in steady state in a stellar atmosphere?

Recent empirical data provide motivation for this study. Thus, across a certain dividing line among giant stars in the H-R diagram, mass loss seems to become more rapid (Stencel and Mullan 1980), while X-ray emission essentially disappears (Ayres *et al.* 1981). Since solar data indicate that strong X-ray emission is associated with closed magnetic loops, while mass loss is permitted only along open field lines, it is tempting to ascribe the empirical dividing line in the H-R diagram to a change from closed to open magnetic field topology (Mullan 1981, 1982).

The fundamental process in the present discussion is the mechanism by means of which fields evolve from closed to open. Fields begin their evolution in a stellar atmosphere as closed lines, with footpoints rooted in the star, but if mass loss is to occur, the link with the star must eventually be severed. The severing process probably involves more than a single stage. Thus, a loop may undergo MHD evolution at first, while finite resistivity and plasma effects may not play important roles until later in the evolution. In the absence of detailed knowledge of the latter effects in stellar atmospheres, we here confine attention to the first stage of evolution, in which ideal MHD modeling turns out to be adequate to allow

us to draw an important conclusion concerning the ability of closed field lines to persist.

A preliminary answer to the main question posed above can be deduced on the basis of Pneuman's (1968) analytic work on static helmet streamers. This work is summarized in § II. A more general MHD treatment of the problem is undertaken here. The method is described in § III. Results for stars of two different gravities are described in § IV. We find some numerical differences from the preliminary conclusions, but the preliminary conclusions are not radically altered. Implications for stellar mass loss are briefly enumerated in § V where we confirm the earlier suggestion that the empirical dividing line involves a transition in magnetic topology. In § VI we examine the dividing line from a different standpoint (namely, breakdown in magnetostatic equilibrium), and we find corroboration for the magnetic topology transition interpretation. Conclusions of the work are presented in § VII.

### II. PNEUMAN'S WORK ON STATIC HELMET STREAMERS IN THE SUN

The magnetohydrostatic equilibrium of a helmet streamer (consisting of closed field lines below, surrounded by open field above) was studied analytically by Pneuman (1968). Various simplifying assumptions were made, including isothermality and a total area function (equal to the sum of closed and open field

# ENERGY DYNAMICS IN STRESSED MAGNETIC FIELDS: THE FILAMENTATION AND FLARE INSTABILITIES

G. VAN HOVEN, R. S. STEINOLFSON, AND T. TACHI

Department of Physics, University of California

Received 1982 September 8; accepted 1982 November 3

## ABSTRACT

The thermal and tearing instabilities are believed to be the two primary temperature modification mechanisms in sheared astrophysical magnetic fields. The former gives rise to the formation of cool filaments and the latter to the release of magnetic energy. It has long been known that these processes are interrelated, most conspicuously in the case of the solar corona where prominences often precede flares within the same magnetic structure. It is also clear, from first principles, that the energy transport underlying the thermal instability should have a strong effect on the resistivity which facilitates magnetic tearing, and that the energy release of the latter should affect the temperature drop of the former. This paper describes some results of the first calculations which attempt to unify the dynamic treatment of these two coexisting instabilities. Growth rates as a function of resistivity, and examples of the primary mode structures are provided, along with a discussion of some critical aspects of the interaction of these two astrophysical energy flux mechanisms.

*Subject headings:* hydromagnetics — Sun: flares

## I. INTRODUCTION

It is a familiar fact, as observed in the solar example, that increasing magnetic field shear or stress gives rise to the formation of filaments (Chiuderi and Van Hoven 1979) and to flares (Van Hoven 1979). Theories of the former mechanism (Field 1965), which is driven by a radiation output that decreases with temperature, have usually ignored the resistive magnetohydrodynamic effects of the resulting, very collisional, relatively low-temperature plasma. Treatments of the latter instability (Furth, Killeen, and Rosenbluth 1963), which is catalyzed by finite conductivity, have usually ignored the energy flux consequences of the resulting magnetic energy release.

Since, in astrophysical situations, radiation is a strong effect and the relevant resistivity is the temperature-dependent Coulomb value, these two dynamic processes should be treated in a unified way. This paper represents the initial step in such an investigation, which will eventually describe the nonlinear development of the dynamics of a resistive, energy-transforming, plasma region subjected to magnetic stress. The present work concentrates on the numerical solution of the linear instability problem, which has also been treated by an analytical method (Steinolfson 1983) that is able to predict approximate growth rates.

## II. FORMULATION

We consider a current-carrying, stationary volume of plasma whose magnetic field is described by the model form

$$B_0 = B_0 [e_x \tanh(y/a) + e_z \operatorname{sech}(y/a)] = B_0 e_h, \quad (1)$$

which is force-free and thus consistent with an initially uniform temperature and density. We describe the development of this system by using the resistive MHD equations, relating flow velocity to magnetic induction, along with the incompressible energy transport equation,

$$K\rho \frac{dT}{dt} = (\gamma - 1) [H(\rho) + \eta(T) J^2 - \rho^2 \Phi(T) + \nabla \cdot \kappa_{\parallel}(T) e_h e_h \cdot \nabla T], \quad (2)$$

specifying the variation of temperature. Here the notation is the same as that used in Chiuderi and Van Hoven (1979), except that  $K$  is Boltzmann's constant per unit (average) mass, an Ohmic heating term is added, and the temperature-dependent Coulomb values (Spitzer 1962) of resistivity  $\eta$  and (dominant) parallel thermal conductivity  $\kappa_{\parallel}$  are used.

## TURBULENT RESISTIVE HEATING OF SOLAR CORONAL ARCHES

GREGORY BENFORD

Physics Department, University of California, Irvine

Received 1982 April 5; accepted 1982 December 3

### ABSTRACT

Field-aligned currents in coronal arches can heat the plasma through instabilities if the electron drift velocity is large enough. We consider the heating as a time-dependent problem in a plasma cylinder, including heating rates for ions and electrons, longitudinal conduction and convection, radiation, and cross-field transport, all using both Coulomb and turbulent effects. Electron drift velocity is held constant, resembling an arch with high inductance. The electrostatic ion cyclotron instability proves the dominant effect at all times, starting from an initially isothermal plasma at  $T = 10$  eV. Ion acoustic modes never become unstable, since  $T_e/T_i$  never exceeds 3. Rapid heating to temperatures of 100–1000 eV occurs, after which a slow “percolation” sets in, with turbulence switching on and off rapidly, keeping the plasma temperature in the  $\sim 100$  eV range. This steady tradeoff between weak microturbulence and macroscopic loss processes determines the long time heating level. Turbulence is described with a resonantly broadened saturation model which enjoys some experimental support. Radiation losses are never important. High density arches heat more slowly ( $\sim 10$  s). Finite plasma pressure effects and turbulent cross-field transport have little influence on the long term temperature. Cases with a turbulent Ohm’s law (low inductance case) also yield strong heating rates. Despite its intrinsic unsteady behavior, resistive heating seems capable of providing efficient arch heating.

*Subject headings:* hydromagnetics — plasmas — Sun: corona — turbulence

### 1. INTRODUCTION

Coronal heating models depend either upon energy deposition by waves traveling up from the base of a coronal loop, or upon currents running from one foot of a loop to another. This paper studies the latter possibility, using anomalous Joule heating by electrostatic ion cyclotron waves.

In the wave model it is difficult to allow for boundary losses and convective effects. In the current-generated turbulence picture, however, careful balancing of losses with turbulent heating rates can be done. (Tucker 1973; Rosner, Tucker, and Vaiana 1978; Nolte, Petresso, and Gerassimenko 1979; Vlahos 1979). Earlier work often assumed that Buneman or ion-acoustic modes were excited in the loop (Hinata 1979). Typically these waves heat the coronal plasma very quickly, produce high resistivity, and lead to large changes in transport phenomena in a very short time ( $< \text{sec}$ ). This may be relevant to strongly driven currents and flares, but it seems doubtful that quiescent heating follows this picture.

Here we study current-driven heating without making any assumptions about which mode is dominant. Instead, we begin with a plasma in a cylinder with  $T_e = T_i$ , and turn on an electron drift at time  $t = 0$ . The ends and sides of the cylinder are held at a constant temperature

(usually  $10^5$  K). We allow currents along  $z$ , the direction of the magnetic field, and further allow the plasma cylinder to expand radially (due to finite plasma pressure). We assume as a basic condition that the self-fields from these heating currents are negligible. A specific picture of how this could be so, Figure 1, portrays currents which reverse direction on scales of about 100 m. This minimizes the energy lodged in self-fields. Our calculation includes turbulent transverse transport, longitudinal conduction, convection, and radiative losses, based on existing laboratory and theoretical work (see § III). Hinata (1980) wrote down similar equations and speculated on possible solutions.

In the absence of inhomogeneities in the equilibrium, there are no unstable drift modes. Because the plasma in the loop starts out isothermal, the first instability excited by relative electron-ion drift is the electrostatic ion cyclotron mode, denoted IC. The critical electron drift speed for exciting this instability is  $u_c = 0.3 (T_i/T_e)^{3/2} v_e$ , which is always less than the electron thermal velocity. In all our solutions, the equivalent electric field driving the currents never exceeds the runaway Dreicer field, so we can use drifting Maxwellian distributions throughout the temporal evolution. The ion cyclotron instability differs from the more virulent ion acoustic and Buneman modes in that it is in principle self-regulating, since  $u_c$  is proportional to  $T_i^{3/2}/T_e$ . Typically, in times less



## MICROWAVE SIGNATURES FROM A RECONNECTING PLASMA PINCH, WITH APPLICATION TO LOOP FLARES<sup>1</sup>

YUNG MOK

Department of Physics, University of California, Irvine  
Received 1983 February 25; accepted 1983 May 19

### ABSTRACT

We calculate the microwave signature of a cylindrical plasma pinch undergoing magnetic reconnection, a process which occurs in many astrophysical situations, such as solar flares. Depending on the viewing angle and the average energy of the accelerated electrons, the microwaves from this betatron-like source show various amounts of circular polarization. The degree of polarization is shown to be frequency dependent, and the sense of polarization is sometimes reversed. The power spectrum is predicted to have several interesting properties, which can be compared with high-resolution measurements.

*Subject headings:* polarization — Sun: flares — Sun: radio radiation

### 1. INTRODUCTION

Microwave radiation is observed in a variety of astrophysical situations. Among many possibilities, such emission can come from energetic electrons traveling in a magnetic field, producing gyrosynchrotron radiation in the microwave range. The origin of these electrons, however, is less well understood. Depending on the physical conditions, there can be a number of acceleration mechanisms, due to turbulence, DC electric fields, etc. Various types of these energization processes have been used to provide microwave emission models.

Generally speaking, for an observed gyrosynchrotron source, a particular energetic-electron model can explain certain characteristics of the microwave spectrum. A more complete theory, however, must also consider the origin of these electrons and the mechanism that accelerates them in the plasma environment and magnetic structure derived from measurements. Since field reconnection instabilities occur in numerous astrophysical conditions, it seems natural to investigate how they give rise to energetic electrons and to calculate their microwave signatures, so that they may be compared with observations.

In the present work, we examine a specific example based on magnetic field reconnection in a magnetized plasma pinch (Mok and Van Hoven 1982). In this configuration, the instability produces a quasi-DC electric field (Van Hoven 1979) in the azimuthal direction, which happens to be parallel to the (azimuthally) closed magnetic field lines. The acceleration is, therefore, extremely efficient so that the effective radial size of the plasma column does not have to be large. In fact, this can be one of the most compact microwave sources in the domain of astrophysics.

To cite a specific application, we choose a simple bipolar magnetic loop, with physical parameters corresponding to those observed in solar active regions. The triggering of a solar flare is considered as the onset of the magnetic reconnection instability. The instability criterion and a detailed analysis of such a physical process have been published (Mok and Van Hoven 1982). In this paper, we will confine ourselves to the emission of microwaves as a consequence of the reconnection process. Although we use a solar flare as an example in the present computation, the principles and methods can be applied to various astrophysical objects under appropriate conditions.

Microwave emission in solar flares has been widely observed and theoretically interpreted (Marsh, Zirin, and Hurford 1979; Marsh *et al.* 1980, 1981; Marsh and Hurford 1980; Alissandrakis and Kundu 1978; Svestka 1976; Scalise, Basu, and Marques Dos Santos 1971; Tanaka and Enomé 1970; Sturrock 1980; Emslie and Vlahos 1980; Dulk, Melrose, and White 1979; Mätzler 1978; Böhme *et al.* 1977; Kundu, Velasamy, and Becker 1974; Takakura and Scalise 1970; Holt and Ramaty 1969; Ramaty 1969). In this work, we calculate the microwave signatures of a reconnecting magnetized plasma pinch and suggest a plausible alternative to the existing loop flare models based on physical conditions in the solar corona. When compared with the location of the microwave source, the polarizations, and the radiated power, our results seem to be in agreement with observations. Since high (frequency) resolution measurements

<sup>1</sup>UCI Technical Report, No. 83-10.

# Energetics and the resistive tearing mode: Effects of Joule heating and radiation

R. S. Steinolfson

*Department of Physics, University of California, Irvine, California 92717*

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Hydromagnetic theory is used to investigate the influence of Joule heating and temperature-dependent radiation on the linear development of the incompressible, resistive tearing instability in both force-free and neutral-sheet magnetic fields in a slab geometry. A temperature-dependent resistivity couples the energetics and the dynamics of the instability. Analytical expressions are derived for the growth rates utilizing, in separate analyses, the constant- $\Psi$  and the long-wavelength (nonconstant- $\Psi$ ) approximations. The solutions indicate the occurrence of several modes in addition to the usual tearing mode. A number of the modes have relatively slow, complex growth rates. However, at large values of the magnetic Reynolds number  $S$  ( $S \gtrsim 10^7$ , the actual value depending on the relevant parametric values), there are at least two modes with purely exponential growth (real, positive growth rates) when the radiative loss decreases with increasing temperature. One is the resistive tearing mode, and the other is a more rapidly growing mode with a growth rate resembling that of a thermal instability. If the radiation is neglected, the Joule heating alone also results in two modes with real, positive growth at large  $S$ . One mode is again the tearing mode, but, in this case, the second mode is a more slowly growing Joule-heating mode. Another point of departure from previous theoretical results is that, below a particular value of  $S$  ( $S \lesssim 10^6$ , the value once again being determined by the parameters), all the modes are generally stabilized.

## I. INTRODUCTION

Resistive tearing instabilities have been invoked as a key constituent in several models of various naturally occurring phenomena in solar-flare physics<sup>1-3</sup> and magnetospheric physics<sup>4</sup> (e.g., substorms, geomagnetic tail) and have been observed in numerous laboratory devices, particularly in tokamak discharges.<sup>5</sup> The common occurrence of this instability and its potential importance in understanding phenomena in which it may occur have prompted a large theoretical effort, initiated by the work of Furth, Killeen, and Rosenbluth<sup>6</sup> (referred to hereafter as FKR). The tearing instability has the potential of occurring at locations where a component of the magnetic field reverses direction. It results in a reconnection of the field lines about the initial location of field reversal with a subsequent conversion of magnetic energy in the initial field to heating and particle acceleration.

Despite the aforementioned and well-recognized<sup>2,3</sup> close connection between the dynamics and energetics of resistive tearing, the linear dynamic evolution of the mode, which is of interest here, has previously been uncoupled from the thermodynamics. This is accomplished by assuming either that the resistivity is constant<sup>1,7-10</sup> or that only convective effects in the energy equation are important.<sup>6</sup> DiBiase and Killeen<sup>11</sup> do include a variable resistivity and a complete energy equation in their numerical study, but they do not include radiation and do not attempt to isolate the effect of the other energy term in which we are interested: Joule heating. Studies of the thermal instability<sup>12,13</sup> do include the energy equation with the heating and radiation, but they assume zero resistivity, and hence, do not allow for the competing development of the tearing instability.

We have undertaken a study to investigate the contribution of energy flux to the dynamics of magnetic field reconnection, with the aim of unifying the resistive tearing<sup>1</sup> and thermal instability<sup>13</sup> mechanisms in a sheared magnetic field. We are pursuing this study from two separate, but complementary, approaches—the primary one being numerical<sup>14,15</sup> and the other, as in the present paper, analytical. Our model includes the effects of resistivity in Ohm's law, and Joule heating, radiation, and thermal conduction in the energy equation. The numerical approach solves the model equations without any further approximations beyond linearization. The analytical approach, on the other hand, requires additional approximations and, as presented herein, is only valid for negligible thermal conductance.

A temperature-dependent, Coulomb-like resistivity is considered in the present paper, thereby providing the coupling between the dynamics and the energy equation. In thermal instability studies the coupling is accomplished by compressibility. However, as is commonly done in tearing-mode analyses, the plasma is here assumed to be incompressible. We use magnetohydrodynamic (MHD) theory with Joule heating and temperature-dependent radiative loss included in the energy equation.

Our object is to determine the influence of Joule heating and radiation on the linear development of the tearing instability in slab geometry. The analysis closely parallels that of FKR for modes which satisfy the constant- $\Psi$  approximation. For long-wavelength modes which do not satisfy constant  $\Psi$ , the approach of Coppi *et al.*<sup>9</sup> (hereafter cited as CGPRR) and Pritchett *et al.*<sup>10</sup> is followed. The differentiation between constant- $\Psi$  and nonconstant- $\Psi$  solutions

# The effects of Ohmic heating and stable radiation on magnetic tearing

T. Tachi, R. S. Steinolfson, and G. Van Hoven

Department of Physics, University of California, Irvine, California 92717

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A study is made of the effect of a temperature-dependent Coulomb-like resistivity on the planar tearing mode. The local evolution of the temperature is described by an energy equation which includes Joule heating and optically thin radiation. The resulting system of coupled linear magnetohydrodynamic equations is solved numerically, and eigenfunctions and growth rates are obtained. In the absence of radiation, there are two distinct solutions above a critical value of the magnetic Reynolds number  $S$ , a tearing-like mode and a Joule-heating mode. Below this point, the growth rates coalesce into a conjugate-complex pair. When stable radiation ( $dR/dT > 0$ ) is added, the heating mode disappears and a modified tearing excitation exists to much lower values of  $S$  before its growth is cut off by Ohmic heating. Examples are given for solar coronal parameters, and for those characteristic of fusion-research devices. The introduction of an effective value for the resistivity, in the presence of energy transport, allows a simple qualitative discussion of the different modes.

## I. INTRODUCTION

The resistive tearing mode has been a topic of considerable interest since it was first proposed by Furth, Killeen, and Rosenbluth<sup>1</sup> (FKR). It has been the subject of linear analysis, showing two principal regimes of interest, the "constant- $\Psi$ " case<sup>1</sup> and the small-wavenumber limit,<sup>1,2</sup> and an intermediate range where the growth rate is maximum.<sup>3</sup> Connections between these regimes have been made<sup>4,5</sup> which show the relationship between the planar and cylindrical forms, and their nonlinear behavior has been examined.<sup>6-9</sup>

The essence of the physical cause underlying magnetic tearing is the spontaneous generation, in a sheared field, of a very narrow layer where accelerated resistive diffusion, or field reconnection, can occur. The rate of this process depends directly on the value of the resistivity  $\eta$  in the tearing layer, scaling as  $\eta^\nu$  where  $0.4 < \nu < 0.67$ . It is a fact that the relevant resistivity, in most astrophysical and laboratory situations, is the Coulomb value<sup>10</sup>  $\eta = \eta_c T^{-3/2}$ , and that Ohmic diffusion heats the ambient plasma. Thus, the question arises whether the localized Joule heating, caused by the reconnection, can quench the instability. When one examines previous attacks on this problem, one discovers instances where a variable resistivity has been included in the formulation,<sup>1,8,9</sup> but no cases where its dynamic effects have been carefully studied. Almost all previous calculations assume constant resistivity.

In the present paper we examine the consequences of the use of a space-time-variable Coulomb resistivity to catalyze magnetic tearing. In order to describe the necessary temperature evolution, we use an incompressible (slow) form of the energy-transport equation, which includes the effects of Ohmic heating and its principal energy-loss competitor, optically thin radiation. The emission laws used have positive temperature slopes (e.g., bremsstrahlung), so that we need not consider radiatively unstable modes.<sup>11,12</sup> Finally, we do not include thermal-conduction losses,<sup>11</sup> since they are negligible at the relatively small wavenumbers which are relevant for magnetic tearing. A recent publication on the

normal tearing mode,<sup>5</sup> a brief report on the full energy-transport formulation including unstable radiative modes,<sup>12</sup> and an analytical treatment,<sup>13</sup> which predicts the growth rates found here, serve as an immediate background to the present work. The latter paper<sup>13</sup> uses the constant- $\Psi$  and small-wavenumber approximations to show the existence of the tearing, radiative, and Joule-heating modes.

In the following sections we describe the formulation and the numerical methods (II), and the results of the computations (III), including both solar-coronal and fusion-research examples. Section IV contains a discussion of the energy dynamics in terms of an "effective" value for  $\eta$  and the conclusions derived from this study.

## II. EQUATIONS

We consider a stationary distribution of plasma with uniform density and temperature throughout, the latter maintained against radiation by a constant heat input. The equilibrium plasma is thus force-free (has no pressure gradients) and its magnetic field is described by

$$\mathbf{B}_0(y) = B_0 [\hat{e}_x \tanh(y/a) + \hat{e}_z \operatorname{sech}(y/a)], \quad (1)$$

where  $\hat{e}_x, \hat{e}_z$  is a unit vector in the  $x(z)$  direction and  $a$  specifies the magnetic shear scale.

It is then assumed that the magnetohydrodynamic equations properly describe the evolution of the plasma in terms of its magnetic field  $\mathbf{B}$ , its flow velocity  $\mathbf{v}$ , and its temperature  $T$ , and that the plasma is incompressible, inviscid, and free from gravitational effects.

The equations in cgs units are given by

$$\nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{B} = 0, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{c^2}{4\pi} \nabla \times [\eta(T)(\nabla \times \mathbf{B})], \quad (3)$$

$$\frac{\partial}{\partial t} (\nabla \times \mathbf{v}) = -\nabla \times (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{4\pi\rho_0} \nabla \times [(\nabla \times \mathbf{B}) \times \mathbf{B}], \quad (4)$$

$$\frac{\partial T}{\partial t} = \frac{\gamma - 1}{2k_b n_0} [nJ^2 - n_0^2 \Phi(T) + H(n_0)], \quad (5)$$

## RADIATIVE TEARING: MAGNETIC RECONNECTION ON A FAST THERMAL-INSTABILITY TIME SCALE

R. S. STEINOLFSON AND G. VAN HOVEN  
Department of Physics, University of California, Irvine  
Received 1983 February 10; accepted 1983 June 8

### ABSTRACT

Two energy modification mechanisms which are known to occur in sheared magnetic fields are the tearing and thermal instabilities. These processes can be studied separately with formalisms incorporating just the effective driving mechanism of interest (finite resistivity for the tearing mode and unstable radiation for the thermal mode). A model which includes both effects, and a temperature-dependent resistivity, indicates that modified forms of these two instabilities may coexist for identical physical conditions. When they are isolated computationally, one can show that their limiting growth rates are approximately those of the uncoupled instabilities. The spatial structure and energy content of these two new hybrid processes are then individually examined and are found to differ considerably from those obtained from separate treatments of the driving mechanisms. The faster radiative instability, which has a hydromagnetically scaled growth rate like the condensation mode of the thermal instability, is shown to involve a substantial amount of magnetic field reconnection. This can be partially explained by a large temperature drop (or resistivity rise) at the X-point. The island width of the Coulomb-coupled radiative mode is 30% of that produced by a comparable level of the slower tearing instability. In addition, the perturbed magnetic energy in the radiative instability is 5 times that of the perturbed thermal energy, indicating an appreciable modification of the initial magnetic structure.

*Subject headings:* hydromagnetics — instabilities — Sun: flares

### 1. INTRODUCTION

The resistive tearing instability (Furth, Killeen, and Rosenbluth 1963) has long been recognized as a potential candidate for initiating the release and conversion of magnetic energy in sheared astrophysical fields. (See, for instance, Sturrock 1968; Coppi and Friedland 1971; Van Hoven and Cross 1973). For the solar flare application of interest here, one shortcoming of the original tearing instability is that the rate of energy conversion, based on observed large-scale quantities in the solar atmosphere, is somewhat slow. The predicted time scale can be shortened by using small shear scales (below the observable spatial resolution) or unexpected combinations and values of the thermodynamic quantities and magnetic field strength. Additionally, shorter time scales can be obtained from long-wavelength disturbances (Van Hoven 1976; Steinolfson and Van Hoven 1983), or by hypothesizing the existence of either multiple, interacting, tearing layers (Pritchett, Lee, and Drake 1980), or the onset of micro-instabilities near the tearing layer (Coppi and Friedland 1971).

We suggest that the time scale of the instability and, therefore, of the energy release may be reduced by approximately two orders of magnitude in solar atmospheric conditions by recognizing that the conversion may occur in a radiatively modified variation of the (dynamic) tearing instability introduced by Furth, Killeen, and Rosenbluth (1963). We propose that the energy conversion takes place in an instability which involves tearing but which has been substantially altered by energy transport, in particular, by Joule heating and unstable (emission varies inversely with temperature) optically thin radiation. This instability, called the radiative instability, is one of two purely growing modes

that coexist when the energetics are coupled to the dynamics of the tearing mode. The existence and structure of the two modes have been described by Van Hoven, Steinolfson, and Tachi (1983) and their growth rates given in analytic form by Steinolfson (1983). As demonstrated in these studies, the growth of the faster radiative mode is similar to that of the condensation mode of the thermal instability (Field 1965) in the ideal magnetohydrodynamic limit, in a sheared field (Chiuderi and Van Hoven 1979), while the growth rate of the slower mode is similar to that of the usual tearing mode (Furth, Killeen, and Rosenbluth 1963; Coppi *et al.* 1976; Steinolfson and Van Hoven 1983) as obtained without inclusion of the energetics. Although the growth rate of the radiative mode is determined by the unstable radiation, it has not previously been established that it also involves appreciable reconnection at the same rate. It could have been that the radiative instability just produced alternating hot and cold spots (relative to the background), without any substantial reduction in the magnetic energy, as in the conventional thermal instability.

Previous studies of the radiative instability have concentrated on determining when purely growing modes exist, their growth rates, and their eigenfunctions. We now extend this study and make a detailed comparison of the spatial structure and the energy content of the two instabilities (tearing and radiative). In order to accomplish this in a meaningful manner, the disturbance amplitude of each mode must be selected such that the two modes are effectively equal in some sense. The relative amount of tearing and reconnection in the two modes depends, of course, on the method used to select the amplitudes of the disturbances. We compare the modes when the total contribution from the nonlinear

## RADIATIVE AND RECONNECTION INSTABILITIES: FILAMENTS AND FLARES

G. VAN HOVEN, T. TACHI, AND R. S. STEINOLFSON

Department of Physics, University of California, Irvine

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### ABSTRACT

An investigation of the energetically coupled tearing and radiative instabilities in sheared magnetic fields has been made. It has been reported elsewhere that, if a temperature-dependent Coulomb-like resistivity is used in the normal tearing-mode analysis, instead of a constant resistivity, it is possible to study the local evolution of the temperature in terms of an energy equation. When the energetics further include Joule heating and optically thin unstable radiation, their effect on the dynamics of the instability produces a pair of purely growing unstable modes. One is the usual tearing mode, and the other is the radiative mode, which exhibits growth similar to the well-known condensation mode of the thermal instability in ideal compressible MHD. The modes are distinctly separated at high values of the magnetic Reynolds number  $S$ , but, as this parameter is lowered, a mutual coupling between the modes appears, and the two growth rates finally merge at a critical value of  $S$ , below which no purely growing mode exists. This quenching of the unstable modes is shown to be strongly wavenumber dependent. A study of the eigenfunctions and their derived characteristics shows that the radiative mode exhibits a large temperature drop in the reconnection region, due to the overwhelming dominance of radiative cooling over Joule heating, whereas the tearing mode exhibits a much smaller drop in the central layer. The strong X-point resistivity increase, connected with the radiative-mode temperature perturbation, helps to explain the significant magnetic reconnection and field perturbation provided by this mode. For typical solar coronal conditions, the growth rate of the radiative mode is approximately two orders of magnitude larger than the growth rate of the tearing mode. Thus, the time scale of the energy conversion due to magnetic reconnection can be greatly reduced by the radiative mode.

*Subject headings:* hydromagnetics — instabilities — Sun: corona — Sun: flares

### 1. INTRODUCTION

The energy source of a solar flare is generally believed to reside in the strong, stressed, coronal magnetic fields existing above photospheric active regions. The magnetic tearing instability (Furth, Killeen, and Rosenbluth 1963) has been proposed as the mechanism for the release of this stored energy (Sturrock 1968; Coppi and Friedland 1971; Van Hoven 1976, 1979). This resistive process allows the magnetic energy to be dissipated directly into particle acceleration and heating by collision-dominated magnetic field reconnection. Despite the fact that the dynamics and energetics of the instability are thus coupled together, the assumption of a constant resistivity in an incompressible plasma, which has been used in most tearing-mode analyses (Furth, Killeen, and Rosenbluth 1963; Cross and Van Hoven 1971; Coppi *et al.* 1976; Pritchett, Lee, and Drake 1980), does not allow a simultaneous study of the two important aspects of the instability, namely, the dynamics of reconnection and the energetics of Joule heating and radiation. Whereas the use of a Coulomb-like collisional resistivity (Spitzer 1962) allows us to study the dynamics and energetics of the tearing mode, the particular temperature dependence of the resistivity suggests the possibility that local heating may introduce a self-quenching effect on the instability, thus raising a question as to its adequacy in modeling a solar flare.

The first consistent study of the effect of the collisional temperature-dependent resistivity has been approached numerically by Van Hoven, Steinolfson, and Tachi (1983) and analytically by Steinolfson (1983). These reports briefly described the existence of a number of unstable modes, along

with a comparison of their growth rates and eigenfunction profiles, which appear in the different cases where the energy equation contains (1) Joule heating only, (2) Joule heating and stable ( $d/dT > 0$ ) optically thin radiation, (3) Joule heating and unstable ( $d/dT < 0$ ) radiation, and (4) the additional influence of the parallel (to  $B$ ) component of thermal conduction. Cases 1–3 were treated with computational and analytical methods, while case 4 was studied numerically. The analytic approach considered both force-free and neutral-sheet magnetic configurations, while the computational approach concentrated on force-free equilibria.

An extensive numerical study of cases 1 and 2 has been carried out by Tachi, Steinolfson, and Van Hoven (1983), who showed that, in the absence of temperature-dependent radiative power loss, there exists a pair of unstable modes (tearing-like and Joule-heating) which, below a certain low value of  $S \approx 10^6$ , coalesce and become (slowly) overstable as a result of a mode coupling, so that no purely growing unstable mode remains. When stable ( $d/dT > 0$ ) radiation is added, for the particular set of initial parameters chosen for the study, the effect is to remove the Joule-heating mode and to allow the tearing-like mode to persist to a much lower value of  $S \approx 10^2$ . This change results from the suppression, by the very effective radiative energy loss, of the temperature rise due to Joule heating. The mutual coupling and quenching of the mode, at low  $S$ , appears when Joule heating overwhelms radiation.

The results from the study of cases 3–4 with unstable ( $d/dT < 0$ ) radiation were nearly identical since, in the latter case, only a long-wavelength limit was considered, and the

# Nonlinear evolution of the resistive tearing mode

R. S. Steinolfson and G. Van Hoven

Department of Physics, University of California, Irvine, California 92717

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The nonlinear behavior of the tearing instability is investigated with numerical solutions of the resistive, incompressible, magnetohydrodynamic equations. Simulations have been completed for values of the Lundquist number  $S$  from  $10^2$  to  $10^6$  and wavelength parameter  $\alpha = 2\pi a/\lambda$  from 0.042 to 0.5. The initial state for the nonlinear computations is provided by the linear instability. In all cases, the nonlinear mode initially evolves at the linear rate, followed by a period of considerably reduced growth. At high  $S$  and low  $\alpha$ , secondary-flow vortices, opposite in direction to the linear vortices, generate a new magnetic island centered at the initial X point. The nonlinear growth rate of the one constant- $\psi$  (in the linear regime) solution considered is approximately an order of magnitude less than that of a comparable nonconstant- $\psi$  solution over the same time period. The nonconstant- $\psi$  computations indicate a reduction of from 8% to 27% of the initial shear-layer magnetic energy, with the larger reductions occurring for the longer wavelength disturbances. The constant- $\psi$  simulation shows a reduction two orders of magnitude smaller. The island width of the nonconstant- $\psi$  solution becomes larger than twice the scale of the initial shear layer. For all cases, the electric field parallel to the magnetic field grows, at the end of the run, to about  $10^{-3}$  times the Drieger field.

## I. INTRODUCTION

Ambient plasma and magnetic field configurations, undergoing relatively slow temporal changes, are often susceptible to purely growing instabilities, on a much shorter time scale, in response to small-amplitude disturbances. Linear theory can frequently be used to determine the parameter space in which a particular instability exists as well as the initial growth rate. As useful as the linear results are, they are seldom capable of establishing whether the instability may ultimately be responsible for the large-amplitude, observable consequences. Without relying on questionable extrapolations of the linear computations, the only way to obtain this information is through an examination of the subsequent nonlinear evolution.

We consider the nonlinear phase of the resistive tearing mode.<sup>1-3</sup> This instability may be important in astrophysical<sup>4</sup> and magnetospheric<sup>5</sup> phenomena, as well as in laboratory plasmas.<sup>6</sup> Our interest is primarily in solar applications, such as the flare. Previous nonlinear treatments have concentrated on the energy output,<sup>7</sup> special analytic limits,<sup>8</sup> or have been limited either to restricted ranges of parameter space (which are not relevant to the flare) or to multiple layers.<sup>7,9</sup>

Possible changes in the nonlinear structure and evolution of the tearing mode over a range of parametric values are of particular interest. The influence of the linear conditions<sup>3</sup> (e.g., the constancy of  $\psi$ ) on the nonlinear growth characteristics is investigated. We also compute the astrophysically relevant quantities of global energy release and local electric fields, which can provide observable diagnoses of such magnetic reconnection processes.

We study the nonlinear evolution in slab geometry using resistive magnetohydrodynamic (MHD) theory, with the main assumptions being those of incompressibility and negligible viscous dissipation. The initially stationary plasma, with constant thermodynamic properties, is embedded in a

force-free magnetic field. Our approach is to use a linearly evolving mode that has attained its maximum linear growth as the initial state for the nonlinear computation. This minimizes the nonlinear calculations, ensures that the nonlinear phase evolves from a truly linear disturbance, and sets the spatial scale for the simulation. Computations have been carried out for values of the Lundquist number  $S$  from  $10^2$  to  $10^6$  and of the wavelength parameter  $\alpha$  from 0.042 to 0.5.

A principal result of our computations is that the nonlinear evolution generally differs from one region of parameter space to another, and, hence, a prototypical characterization of the evolution in the nonlinear regime is not possible. As an illustration of this, constant- $\psi$ <sup>1,3</sup> linear modes evolve much more slowly and convert considerably less of the stored magnetic energy in their nonlinear phase than do those with nonconstant  $\psi$ <sup>2,3</sup> (long wavelengths). Similarly, high- $S$  ( $\geq 10^3$ ), low- $\alpha$  ( $\leq 0.1$ ) modes undergo a change in configuration space with the formation of a new magnetic island centered at the original X point, while the remaining solutions do not.

Some general characteristics that apply to all solutions are: (1) The nonlinear spatial distributions of the physical variables differ substantially from the linear behavior; (2) once nonlinear effects become important, the growth slows considerably from the linear rate; (3) the growth rate is a poor predictor of the nonlinear performance of a particular mode in terms of magnetic energy conversion; (4) the majority of the stored magnetic energy is resistively converted to thermal energy ( $> 80\%$ ) for the long wavelength modes with  $\alpha < 0.5$ ; and (5) maximum electric fields parallel to the magnetic field are about three orders of magnitude less than the Drieger field.<sup>10</sup>

The computational procedure is outlined in the following section, and the numerical results are presented in Secs. III and IV. The final section includes some elaboration on our results, particularly with respect to solar flares, as well as

## THERMAL RIPPLES IN A RESISTIVE AND RADIATIVE INSTABILITY

R. S. STEINOLFSON

Department of Physics, University of California, Irvine  
Received 1983 October 24; accepted 1983 December 14

### ABSTRACT

The inclusion of parallel and perpendicular thermal conduction in the analysis of the tearing and radiative instabilities produces modified versions of these two modes. This improvement of the energy-transport description also introduces a large number of new instabilities whose growth rates lie between those of the thermally conductive tearing and radiative modes. With the exception of the modified radiative mode, all of the modes with conductive heat transport contain spatial temperature oscillations, perpendicular to the tearing surface, on a scale comparable to the width of the resistive tearing layer. These thermal ripples arise in a region where perpendicular thermal conduction and unstable, optically thin radiation are the dominant energy-transport mechanisms. This region extends from the edge of the inner resistive layer out to a distance sufficiently removed from the tearing surface that parallel thermal conduction once again dictates the energetics in a force-free field configuration. The oscillating temperature in the new, thermally dominated modes produces small-amplitude (relative to the temperature) velocity oscillations. The magnetic field perturbation, however, remains unaltered from its behavior without conductive heat transport and is essentially identical to that obtained in the usual tearing-mode analysis.

*Subject headings:* hydromagnetics — instabilities — Sun: corona

### 1. INTRODUCTION

Sheared magnetic fields, in which one component of the field reverses direction, are susceptible to development of the resistive tearing instability. This has been established with incompressible, resistive magnetohydrodynamic (MHD) theory, which considers the dynamic magnetic field-velocity interaction (Furth, Killeen, and Rosenbluth 1963). In applying this theory to solar coronal plasmas, optically thin radiative energy loss may strongly influence the resistively driven reconnection. For instance, it has been shown that the same sheared, force-free, magnetic field configurations which are amenable to resistive tearing also create a unique spatial location for the occurrence of a radiatively driven thermal instability (Field 1965; Chiuderi and Van Hoven 1979). Resistivity (and, therefore, reconnection) was omitted from these compressible MHD analyses, the essential physics being provided by the velocity-energy (temperature) interaction.

More recently, the competing effects of field reconnection and radiative loss were combined into a unified theory by Van Hoven, Steinolfson, and Tachi (1983) and Steinolfson (1983). Two unstable, purely growing modes resulted from this study. The faster radiative mode is predominantly radiation driven and has a growth rate typical of the uncoupled condensation mode of the thermal instability. The mainly reconnection-driven tearing mode grows like the usual resistive tearing instability. Both, however, differ from their respective uncoupled modes, and each has characteristics of the other. This has been demonstrated by Steinolfson and Van Hoven (1984) who show that the radiative instability involves a level of reconnecting fields comparable to that in the tearing branch.

Thermal conduction has generally been neglected in theories similar to those discussed above—especially when the field is force-free. Parallel conduction was omitted on the assumption that the force-free field would thermally isolate x-points and magnetic islands (also assuming insignificant energy transport to external boundaries parallel to the plane in which reconnection occurs). Parallel thermal conductivity exceeds perpendicular

conductivity by approximately 12 orders of magnitude in the solar atmosphere; hence, in regions where they compete, one would expect parallel transport to dominate (Chiuderi and Van Hoven 1979). These contributions to the thermal conduction become comparable when the cross-field temperature gradient has a scale of 10 cm (for the physical conditions used later in this paper). This is much smaller than the resistive tearing layer width (Furth, Killeen, and Rosenbluth 1963) for the same conditions, which is  $10^3$  cm. Based on this argument, the perpendicular component should be inconsequential in instabilities involving reconnection since the tearing-layer width usually dictates the smallest spatial scale. However, in a force-free magnetic field, parallel conduction near the field-reversal location ( $y = 0$  in the slab geometry discussed later) does not directly contend with the perpendicular component since it is effectively nullified by the large (infinite) distance to the boundaries. As a result, perpendicular conduction then competes with the other prominent energy transport mechanism—radiative loss. These two become commensurate for temperature gradients over a distance of  $1.8 \times 10^4$  cm, which is larger than the tearing-layer width discussed earlier, thereby implying that perpendicular heat flux cannot be neglected. At some larger distance from the tearing layer, the usually dominant parallel conduction again governs the evolution. Thus, contrary to usual expectations, the force-free field configuration creates an environment, near the field reversal, where perpendicular thermal conduction may influence energy transport.

Both parallel and perpendicular thermal conduction are considered in this paper, in addition to finite resistivity and radiative loss. After presenting the equations and the model in the following section, the spatial singularity that arises with consideration of only the parallel heat conduction (no perpendicular component) is discussed in § III. The removal of this singularity and the formation of temperature oscillations (thermal ripples) by inclusion of the perpendicular heat-flux component are demonstrated in § IV. Some elaboration on the

# Resonant absorption of phase-mixed Alfvén surface waves in ideal and resistive magnetohydrodynamics: Initial-value problem

R. S. Steinolfson

Department of Physics, University of California, Irvine, California 92717

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Numerical solutions of the linearized magnetohydrodynamic equations are used to investigate resonant energy absorption in the continuous frequency spectrum. Energy in the phase-mixed surface waves resistively dissipates with the absorption time and width scaling as resistivity to the  $-1/3$  and  $1/6$  powers, respectively.

The phenomenon of Alfvén surface waves in a medium with nonuniform Alfvén velocity is well known and has been studied extensively for some time (see, e.g., Hasegawa and Uberoi<sup>1</sup> and references therein). Early interest in the surface wave centered on low-frequency heating of laboratory plasmas.<sup>2</sup> The highly structured nature of the solar atmosphere subsequently generated interest in these waves as a candidate for the elusive coronal-heating mechanism.<sup>3</sup>

Energy transfer from the surface waves to the plasma is generally considered to involve phase-mixing. As background, it is worth noting that surface waves propagating on a discontinuity are not damped and that the medium, in this case, supports a discrete spectrum of modes. In addition, distinct normal modes on a continuous magnetic field or density profile must oscillate indefinitely (imaginary frequency) in the ideal magnetohydrodynamic (MHD) limit.<sup>4</sup> Surface-wave damping (resonant absorption) occurs as a consequence of the generation of a continuous frequency spectrum by disturbances in a nonuniform, but continuous, plasma. The normal modes of the continuum phase mix and eventually result in a reduction of wave amplitude. The damping is centered at the spatial location (resonance) where the continuum frequency matches the local Alfvén velocity.

Resonant absorption in the continuous spectrum has been treated analytically with the linearized equations of ideal MHD.<sup>5</sup> The solution consists of noncollective oscillatory modes of the continuous spectrum and collective oscillatory modes that are exponentially damped (the resonantly absorbed modes). The further addition of resistivity to this model does not alter the absorption rate of ideal MHD in the limit of small resistivity.<sup>6</sup> Although the wave decay can be computed with ideal MHD, the actual energy absorption must occur by nonideal processes such as the conversion of surface waves to kinetic Alfvén waves<sup>7</sup> or by fluid dissipative processes (e.g., resistivity).

In the present letter we consider an initial-value approach to the study of Alfvén surface waves, in which the linearized MHD equations are solved numerically in time and space. A disturbance of a particular wavelength is excited in a nonuniform plasma, and its temporal evolution is simulated with numerical solutions of the ideal and, separately, resistive MHD equations in slab geometry. This procedure generates a continuous spectrum and, therefore, simulates phase-mixing and subsequent resonant absorption. Resistive dissipation provides the dominant energy absorp-

tion mechanism in this study, rather than mode conversion to kinetic Alfvén waves.

A slab geometry is used with an ambient magnetic field directed along the  $z$  axis and nonuniform in the  $x$  direction. All quantities are assumed independent of the third dimension, and the initial density is taken to be uniform. The linearized, incompressible momentum and induction equations can then be written as (in dimensionless form and for a discrete disturbance wavenumber in the  $z$  direction,  $\exp(ikz)$ )

$$\frac{\partial}{\partial t} \nabla^2 v_x + \alpha F \nabla^2 B_x - \alpha B_x \frac{d^2 F}{dx^2} = 0, \quad (1)$$

$$\frac{\partial B_x}{\partial t} = \alpha F v_x + \frac{1}{S} \nabla^2 B_x, \quad (2)$$

where  $\nabla^2 = \partial^2/\partial x^2 - \alpha^2$ . The  $x$  components,  $v_x$  and  $B_x$ , are given by the solenoidal equations. The quantity  $F$  represents the initial magnetic field, which is assumed to be uniform outside a region of width  $2a$  centered at  $x = 0$ . All distances are referenced to this distance  $a$ . The initial field decreases monotonically from the magnetic field reference value  $B_0$  ( $F = 1$ ) for  $x < -1$  to a smaller value  $B_{\min}$  ( $F = B_{\min}/B_0$ ) for  $x > 1$ . In the nonuniform region  $F$  varies as a fifth-order polynomial of odd powers with zero derivatives at  $x = \pm 1$ . Time is referenced to the Alfvén time  $\tau_a$  and velocities to  $a/\tau_a$ .

The two parameters in Eqs. (1) and (2) are the wavelength parameter  $\alpha = 2\pi a/\lambda$  (wavenumber  $k = 2\pi/\lambda$ ) and the Lundquist number  $S = \tau_r/\tau_a$ , where  $\tau_r$  is the resistive time. The two times are given by  $\tau_a = a(4\pi\rho)^{1/2}/B_0$ ,  $\tau_r = 4\pi a^2/c\eta$ , where  $\rho$ ,  $c$ , and  $\eta$  are the uniform density, light speed, and uniform resistivity, respectively.

Equations (1) and (2) are solved numerically using a finite-difference method that has been successfully applied in the study of resistive tearing modes.<sup>8</sup> The time step must be small enough to follow the temporal oscillations. A nonuniform grid is necessary to resolve the spatial oscillations in the phase-mixed waves and to compute their asymptotic (large  $|x|$ ) behavior with a reasonable total number of grid points.

A velocity disturbance of the form  $v_x(x) \propto \exp(-\alpha|x|)$  initiates the surface waves. This particular disturbance was selected since it excites just the surface wave, while numerous other attempted forms (e.g., a broader or a uniform disturbance) also trigger transverse Alfvén waves in the uniform medium. The boundary conditions require that the perturbed variables in the time-dependent computation have



# Nonlinear computations of a solar flare model

H. Strauss

*Courant Institute for Mathematical Sciences, New York University, New York 10012*

G. Van Hoven

*Department of Physics, University of California, Irvine, California 92717*

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A cylindrical axisymmetric tearing mode model for solar flares is investigated numerically. Large magnetic energy release only occurs when there are at least two mode rational surfaces in the current-carrying plasma.

## I. INTRODUCTION

It has been proposed that solar flares may be produced by reconnection of magnetic flux loops.<sup>1</sup> However, it has been recently pointed out that the finite length of magnetic arch is strongly stabilizing,<sup>2</sup> precluding all but sausage-type ( $m = 0$ ) tearing modes. In this paper we investigate the nonlinear development of  $m = 0$  tearing modes in the cylindrical solar flare model of Mok and Van Hoven.<sup>3</sup> We find that for substantial release of magnetic energy it is necessary to have more than one mode rational surface within the current-carrying plasma.

In a study of linear  $m = 0$  tearing stability, Mok and Van Hoven found that fast reconnection with the appropriate timescale for solar flares occurred when a mode rational surface was located at the edge of the current channel. The nonlinear development of this fast reconnection is quite different, depending on the number of mode rational surfaces present. If there is only one rational surface, the released magnetic energy is less than 1% of the magnetic energy of the initial equilibrium. When there are two mode rational surfaces, the equilibrium current is larger and much more free energy is available during reconnection. The magnetic energy released is a large fraction, about 25%, of the energy stored in the initial equilibrium, as in the planar force-free case.<sup>4</sup>

As part of the nonlinear development, short-wavelength magnetic islands coalesce to form islands with the longest available wavelength. This suggests that the final reconnected state depends primarily on the magnetic forces and is not sensitive to the form and strength of the resistivity.

## II. EQUATIONS OF MOTION AND LINEAR STABILITY

The solar flare equilibrium model of Mok and Van Hoven is a cylindrical plasma of length  $L$ , with a force-free magnetic field. Within the current channel of radius  $r_c$ , the field is given by the well-known Bessel function model of Lundquist.<sup>5</sup> For radii greater than  $r_c$ , the plasma current is assumed to vanish. In our computations, we have caused the current to fall off smoothly to zero in a layer of width  $\delta$ . We have also assumed the presence of a fixed conducting shell of radius  $r_s > r_c$ , for numerical reasons, rather than allowing the fields to extend to infinity.

We have studied the linear and nonlinear, axially symmetric, resistive modes associated with this model, using the incompressible equations of motion<sup>6,7</sup> in cylindrical coordinates  $r, \theta, z$ , with resistivity  $\eta$  and viscosity  $\mu$ ,

$$\mathbf{B} = \nabla\psi \times \nabla\theta + I\nabla\theta, \quad (1)$$

$$\mathbf{v} = \nabla U \times \nabla\theta + K\nabla\theta, \quad (2)$$

$$\begin{aligned} \frac{d}{dt}(\nabla \cdot r^{-2} \rho \nabla U) &= \nabla(r^{-2} \Delta \cdot \psi) \times \nabla \psi \cdot \nabla \theta \\ &+ r^{-4} I \frac{\partial}{\partial z} I - \rho r^{-4} K \frac{\partial}{\partial z} K \\ &+ \frac{1}{2} \nabla \rho \times \nabla v^2 \cdot \nabla \theta + \mu r^{-2} \Delta \cdot (\Delta \cdot U), \end{aligned} \quad (3)$$

$$\frac{d}{dt} \psi = \eta \Delta \cdot \psi, \quad (4)$$

$$\frac{d}{dt} (Kr^{-2}) = \nabla(Kr^{-2}) \times \nabla \psi \cdot \nabla \theta + \eta r^{-2} \Delta \cdot I, \quad (5)$$

$$\rho \frac{d}{dt} K = \nabla I \times \nabla \psi \cdot \nabla \theta + \mu \Delta \cdot K, \quad (6)$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \nabla U \times \nabla \theta \cdot \nabla, \quad (7)$$

$$\Delta \cdot = r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \quad (8)$$

The equations may be made dimensionless by normalizing lengths to  $r_c$ ,  $\psi$  to  $Br_c^2$ , and time to  $\rho^{1/2} r_c^{-1} B_c^{-1}$ . A key parameter for tearing modes is the ratio of resistive decay time to the shear Alfvén period, which we will denote as

$$S = B_c(0) r_c \rho^{-1/2} \eta^{-1}. \quad (9)$$

The energy is the sum of kinetic energy  $KE$  and potential energy  $PE$ , with

$$KE = \frac{1}{2} \rho \iint dz dr r^{-1} [(\nabla U)^2 + K^2],$$

$$PE = \frac{1}{2} \iint dz dr r^{-1} [(\nabla \psi)^2 + I^2],$$

$$\begin{aligned} \frac{\partial}{\partial t} E &= -\eta \iint dz dr r^{-1} [(\Delta \cdot \psi)^2 + (\nabla I)^2] \\ &- \mu \iint dz dr r^{-1} [(\Delta \cdot U)^2 + (\nabla K)^2]. \end{aligned} \quad (10)$$

The energy is conserved, except for dissipation due to resistivity and viscosity. The integration extends over the plasma volume  $r < r_c$ ,  $0 < z < L$ .

## FLARE PRECURSORS AND ONSET

G. Van Hoven\* and G. J. Hurford\*\*

\*Department of Physics, University of California, Irvine,  
CA 92717, U.S.A.

\*\*Solar Astronomy, California Institute of Technology, Pasadena,  
CA 91125, U.S.A.

### ABSTRACT

We report on the progress of a search for precursors that have direct physical connections to the start of subsequent solar flares. The discussion includes recent results at radio, visible, ultraviolet, and x-ray wavelengths, which are relevant to the pre-impulsive (onset) phase. We also relate the aspects of a theoretical scenario, based on magnetic reconnection with transport-coefficient phase changes, for explaining flare onset. The pertinent time scales for pre-impulsive temporal developments are discussed.

### INTRODUCTION

Solar activity encompasses a range of timescales. Overall levels follow the well-known eleven-year activity cycle, and large-scale magnetic features evolve on a timescale of months. The life cycle of an individual active region ranges over many days to a few weeks. Particularly in its growth and mature phase, magnetic flux is often seen to change on a timescale of hours. This flux evolution plays an important role in the destabilization of the quasi-static magnetic fields in the corona, and the subsequent release of energy on a timescale of seconds to tens of seconds in the form of solar flares.

Since the 1970's there has been a growing appreciation of an intermediate timescale of active-region changes, those changes occurring on a scale of minutes to perhaps tens of minutes immediately preceding the start of the impulsive flare energy release. This aspect of solar activity, sometimes denoted as the "onset" phase of a solar flare, is worthy of study for a number of reasons, among which is the insight it can give into the plasma instabilities which result in the impulsive energy release.

The pre-SMM status of our knowledge of solar flares and active regions has been reviewed in the monographs which summarized the results of the Skylab Solar Workshops /1,2/. However, one unavoidable weakness of the film-limited Skylab observations was the paucity of data on preflare conditions. In part to remedy this, the Flare Buildup Study was organized as part of the Solar Maximum Year to help coordinate the observational coverage of preflare conditions. One aspect of the Flare Buildup Study/Solar Maximum Analysis program was concerned with Flare Precursors and Onset (FPO). This paper, which summarizes the results of the FPO aspect of the program, is based on the deliberations of a study-group (D) meeting at Big Bear Solar Observatory in 1983, and the subsequent efforts and reports by its members.

This paper is not intended to be a comprehensive review of what is known about flare precursors and onset. Rather it is intended to highlight some of the recent developments. For a review of the observational status prior to the present solar maximum the reader is referred to reviews by Van Hoven et al. /3/ and Martin /4/. Another current discussion will appear in the proceedings of the 1983-84 Solar Maximum Mission workshops.

Using the terminology suggested by Martin /4/, this group restricted its deliberations to the "distinct" category of preflare events, namely those for which direct physical association with the subsequent flare is implied. Studies such as this are, of course, bedeviled by the issue of what is preflare and what is flare. Our choice has been to consider those active-region phenomena which occur on a timescale significantly longer than the impulsive-phase energy release, significantly shorter than the overall flux-evolution process (which is dealt with elsewhere in this issue), and which have a plausible, direct, physical connection with the subsequent energy release. Under these rules the "flare" begins with the start of the rapid, impulsive-phase, energy release. Put another way, we are concerned with the preflash phase of the flare (or equivalently the onset phase in Sturrock's /1/ terminology).

# RADIATIVE AND RECONNECTION INSTABILITIES: COMPRESSIBLE AND VISCOUS EFFECTS

T. TACHI, R. S. STEINOLFSON, and G. VAN HOVEN

*Department of Physics, University of California, Irvine, CA 92717, U.S.A.*

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**Abstract.** Filaments and flares are prominent indicators of the magnetic fields of solar activity. These instability phenomena arise from the influence of weak transport effects (radiation and resistivity, respectively) on coronal magnetodynamics and energy flow. We have previously shown that the filament and flare (tearing or reconnection) mechanisms are resistively coupled in sheared magnetic fields of the kind existing in active regions. The present paper expands this treatment to include the effects of compressibility and viscosity, which are most prominent at short wavelengths. The results show that compressibility affects the radiative mode, including a modest increase of its growth rate, and that viscosity modifies the tearing mode, partially through a decrease of its growth rate. A comprehensive discussion of the mode structures and flows is presented. The strongest effect found is a reversal, at very long wavelengths, of the radiative cooling of the resistive interior layer of the tearing mode, caused by compressional heating.

## 1. Introduction

Filaments and flares are observed to occur in sheared magnetic field regions in the solar atmosphere (Martin, 1973; Van Hoven *et al.*, 1980). The former are primarily driven by unstable radiation (Field, 1965) and the latter by finite resistivity (Furth *et al.*, 1963). Considerable progress has been made on the quantitative description of these mechanisms: the filamentation (thermal) instability has been treated in a nonuniform magnetic field (Chiuderi and Van Hoven, 1979), and the tearing (reconnection) instability has been extended to extreme astrophysical conditions (Steinolfson and Van Hoven, 1983).

Recently, a unified formulation of the interacting radiative and reconnection processes has been achieved (Van Hoven *et al.*, 1983; Steinolfson, 1983; Van Hoven *et al.*, 1984), which applies in the incompressible and inviscid limit. These works established the existence of two unstable modes which coexist in a force-free solar coronal environment. They also examined the structure of the individual modes in terms of the perturbed physical variables and of the derived ratios between their amplitudes and widths. The latter quantities led to a definition of the physical characteristics which clearly distinguish the two modes.

The radiative mode resembles the thermal instability of ideal MHD, with a growth rate similar to that previously found by Chiuderi and Van Hoven (1979) in the compressible limit. This results from the fact that the resistive effect on the energetics of the plasma is small and, as a result, the inertia term is balanced by the unstable radiation, so that the growth rate is found to be directly proportional to the amount of radiative cooling available. The only significant difference between the ideal and resistive modes is the varying degree of magnetic reconnection in the radiative mode (Steinolfson and Van Hoven, 1984), which is a strong function of the wavenumber. In short, the

## RESISTIVE WAVE DISSIPATION ON MAGNETIC INHOMOGENEITIES: NORMAL MODES AND PHASE MIXING

R. S. STEINOLFSON

Department of Physics, University of California, Irvine

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### ABSTRACT

Numerical solutions of the linearized, resistive MHD equations indicate the presence of two characteristic forms of wave decay on magnetic inhomogeneities. Solutions for relatively long-wavelength disturbances and large values of the resistivity have the behavior of decaying normal modes. In the opposite limits, phase mixing becomes important, with the eventual build-up of large spatial gradients in which the resistive terms dominate. This composite behavior is shown to be conceptually consistent with analytic results, which predict that the complete solution consists of the sum of collective and noncollective contributions. The numerical simulations go beyond analytic theory by defining where each contribution prevails and computing wave decay when phase mixing is effective. The dispersion relation in the normal-mode regime is also determined analytically, with some approximations, and is in good agreement with the numerical solution. The decay time of the normal modes varies with Lundquist number as  $S^{1/6}$ , while the phase-mixed decay time scales as  $S^{1/3}$ .

*Subject headings:* hydromagnetics — Sun: corona

### 1. INTRODUCTION

Of the numerous methods that have been proposed for heating the solar corona (for reviews see Kuperus, Ionson, and Spicer 1981; Wentzel 1981), dissipation of magnetohydrodynamic (MHD) waves propagating on magnetic inhomogeneities remains one of the more promising candidates. Application of the theory to coronal loops suggests that the waves may supply enough energy to attain the required heat input (Ionson 1978; Heyvaerts and Priest 1983; Sakurai and Granik 1984; Hollweg and Sterling 1984).

In general, wave-dissipation studies divide into two separate categories depending on the wave polarization. Most attention has focused on disturbances lying in (and propagating in) the plane of the initial field and inhomogeneity. Heyvaerts and Priest (1983), however, considered waves with a displacement perpendicular to both the initial field and inhomogeneity (shear Alfvén waves). The former (referred to here as plane-polarized waves) may decay by virtue of an imaginary component in the frequency spectrum, while the latter lose energy through a phase-mixing process.

The actual energy-transfer mechanism is poorly understood at present, but it has been suggested that it occurs by collisionless processes arising through the excitation of kinetic Alfvén waves (Hasegawa and Chen 1976) or a Kolmogoroff turbulent cascade to high wavenumbers (Heyvaerts and Priest 1983). The position taken in the present paper is that the dissipation results from collisional MHD dissipative processes (e.g., resistivity). Shear Alfvén waves are not considered, although phase mixing will be shown to be important in planar polarization as well.

It can be shown, using the energy or variational principle, that distinct normal modes on a continuous magnetic field or density profile must oscillate indefinitely in the ideal MHD limit (Bateman 1978). Despite this well-known result, an apparent wave decay in the above case can be obtained analytically from ideal MHD by assuming that the frequency (for a real wavenumber) contains both real (oscillating) and imaginary

(decaying) components (Tataronis and Grossman 1973). We attempt to shed some light on these seemingly contradictory results by showing, first of all, that the imaginary frequency from ideal theory agrees well with that computed numerically using resistive MHD. To illustrate that this agreement may not be fortuitous, the addition of resistivity to the analytic, ideal theory is shown to produce a small correction (depending linearly on resistivity) to the nonresistive solution. The picture thereby produced for this normal-mode behavior (and supported by numerical computations) is that the dissipative terms are indeed small at each instant throughout the disturbance, relative to the other terms, yet they produce a cumulative effect resulting in wave decay.

In addition to decaying normal modes, the resistive solution also contains a regime where a phase-mixing process dominates. This disparity of responses is interpreted to be analogous to the behavior predicted analytically from ideal MHD. The ideal solution consists of noncollective oscillatory modes (at the local Alfvén frequency) coupled with collective oscillatory modes (at an average Alfvén frequency), which are exponentially damped at a uniform rate (Sedlacek 1971; Tataronis 1975). The amplitude of the noncollective oscillations decreases inversely with time, which will be shown later to be significant in comparing these results with the numerical computations, including resistivity, presented herein.

One could also envision that the phase-mixing process in ideal MHD occurs when normal modes of the continuum phase mix (Grad 1969). Similarly, the imaginary frequency or "damping rates" computed without dissipation could be interpreted as coupling rates of these normal-mode motions to the continua.

Two separate approaches to the study of wave decay on inhomogeneities are considered in this paper. First of all, ideal and resistive MHD are used to analytically derive the collective (normal-mode) behavior, referred to above, for long-wavelength disturbances. The resistive MHD equations are then solved numerically in time and space using an initial-

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# PROPAGATION OF ENERGETIC ELECTRON STREAMS IN SOLAR FLARES

YUNG MOK

*Physics Department, University of California, Irvine, CA 92717, U.S.A.*

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**Abstract.** The microscopic stability of an electron stream flowing down to the photosphere from the corona is examined. It is found that, while a power-law distribution is stable in the low-density corona, it is unstable against the generation of magnetized electron plasma waves in the high-density photosphere. The scattering of these energetic electrons may alter their radiation signatures.

## 1. Introduction

The generation of energetic electrons, streaming down to the photosphere along a magnetic loop, has been a major feature of many flare models. Although the acceleration mechanism is an open question, observational evidence from X-ray measurements seems to indicate the existence of an electron population with a hard spectrum, leading to thick-target radiation (Jordan, 1981; and references therein). In addition, the energy deposition by these electron streams (beams) has been studied by various authors using different energy spectra, mostly power-law distributions corresponding to observational data (Ricchiazzi and Canfield, 1983; Brown *et al.*, 1978; Lin and Hudson, 1976; Shmeleva and Syrovatskii, 1973; Brown, 1973). However, the question of the propagation of these energetic electrons also deserves careful attention. As a group of energetic electrons streams through a plasma, various interesting phenomena may occur, and eventually affect their propagation characteristics, and their ability to penetrate deep into the photosphere. At this time, it is unclear how such effects will change the electron's ultimate radiation signature.

In the present work, we will study how well such an electron stream can propagate through the solar atmosphere along a magnetic loop. In particular, we will examine their capability for exciting collective instabilities which can significantly change their velocity distribution, giving rise to possibly different radiation signatures. In Section 2, we will study a particular instability which, we believe, is most likely to be excited by an electron stream in this environment. In Section 3, we use a special distribution function model to illustrate this possible scenario. In Section 4, some consequences of this instability are discussed.

## 2. Microscopic Instability

A common feature among all electron-stream (beam) flare models is an anisotropic velocity distribution in the low atmosphere, since an isotropic spectrum cannot carry a net flow. Generally speaking, anisotropic distributions resulting from injection of

# A UNIFIED TREATMENT OF THE FILAMENT AND FLARE INSTABILITIES\*

Gerard Van Hoven  
Department of Physics  
University of California  
Irvine, California 92717 (U.S.A.)

## ABSTRACT

Filaments and flares occur in sheared magnetic structures as a result of radiative cooling and resistive reconnection, respectively. A new integrated theory of these two unstable processes is described, which includes the relevant effects of magnetohydrodynamics and energy transport. The normally dissociated thermal and tearing phenomena are coupled together by a temperature-dependent Coulomb resistivity. As a result, the filamentation and flaring instabilities of a sheared field may coexist, as is familiar from the solar example.

The growth rates and spatial structures of these two modes are detailed here. The much faster radiative instability is shown to provide significant magnetic reconnection, particularly at shorter wavelengths. The long-wavelength reconnection mode is found to be abetted by the resistivity increase caused by the dominance of cooling at the X point, in contrast to its nonradiative behavior. Implications of these results for the development of coronal activity are described.

## INTRODUCTION

It is well known, as exemplified by the development of solar activity, that increasing magnetic-field shear or stress gives rise to the formation of filaments (Chiuderi and Van Hoven, 1979) and to flares (Van Hoven, 1979). Theories of the former mechanism (Field, 1965), which is driven by a radiation output that decreases with temperature, have usually ignored the resistive magneto-hydrodynamic effects of the resulting, very collisional, relatively low-temperature plasma. Treatments of the latter

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Invited paper, to appear in the Proceedings of International Astronomical Union Symposium 107, Unstable Current Systems and Plasma Instabilities in Astrophysics (Reidel).

## NONLINEAR EVOLUTION OF THE RESISTIVE TEARING MODE

R. S. Steinolfson and G. Van Hoven  
Department of Physics  
University of California  
Irvine, California 92717

### ABSTRACT

Numerical solutions of the MHD equations are used to investigate the nonlinear behavior of the tearing instability. The mode evolves from a linearly growing excitation, followed by a period of greatly reduced nonlinear growth. Constant- $\Psi$  solutions evolve much more slowly than comparable nonconstant- $\Psi$  modes with orders of magnitude less conversion of the stored magnetic energy. The nonconstant- $\Psi$  computations indicate a reduction by approximately 20% of the energy in the initial shear layer. For long-wavelength solutions, secondary-flow vortices, opposite in direction to the linear vortices, generate a new magnetic island centered at the initial x-point.

### COMPUTATIONAL PROCEDURE

The nonlinear phase of the tearing mode (Furth et al., 1963) is studied in slab geometry using incompressible, constant-resistivity, MHD theory. The initially stationary plasma, with uniform thermodynamic properties, is embedded in a force-free, nondissipating magnetic field. A linear mode, at its maximum linear growth, provides the initial state for the nonlinear computation. We present results for a magnetic Reynolds number  $S$  (ratio of the resistive time to the hydromagnetic time) of  $10^4$  and values of the wavelength parameter  $\alpha(2\pi a/\lambda)$ , where  $a$  is the shear scale and  $\lambda$  the disturbance wavelength) of 0.05, 0.13, and 0.50. [A larger parameter range and additional computational results are considered by Steinolfson and Van Hoven (1983b).] The mode with  $\alpha = 0.5$  is a constant- $\Psi$  solution in the linear regime, while the other two are nonconstant- $\Psi$ , and the  $\alpha = 0.13$  mode corresponds to maximum linear growth (Steinolfson and Van Hoven, 1983a).

The linear theory predicts a chain of x-points and islands in the magnetic field lying along the tearing surface (x-axis in our geometry) at  $y = 0$ . We isolate one wavelength of this initial disturbance and do

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## Resistive decay of Alfvén waves in a non-uniform plasma

By Y. MOK AND G. EINAUDI†

Department of Physics, University of California, Irvine CA, 92717

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The effect of resistive dissipation on the propagation of an MHD disturbance in a non-uniform plasma is examined. The present analysis, based on a boundary-layer technique, shows the existence of resistive normal modes with complex eigenfrequencies. The real part of the eigenfrequency is associated with an oscillatory behaviour and defines the location in space of the layer where resistivity is important. The dissipation mechanism is responsible for the damping of the wave, in contrast with previous works in which the ideal MHD theory was used.

### 1. Introduction

The properties of magnetohydrodynamic (MHD) waves propagating in a non-uniform plasma have received much attention in the past several years (see Hasegawa & Uberoi 1982, and references therein). This interest arises from the fact that the Alfvén wave is a potential candidate for radio-frequency heating of a fusion plasma and that it can also be responsible, in the highly structured solar atmosphere, for the heating of the magnetized corona.

In the presence of a time-constant, spatially non-uniform, background magnetic field, the infinite-conductivity hydromagnetic wave equation admits solutions which do not have a well-defined spectrum of discrete modes. Thus, no dispersion relation analogous to the uniform case can be found. The reason for this behaviour is that the local variation of the Alfvén-wave velocity gives rise to a singularity in the equations at the point where the wave-phase velocity equals the local Alfvén velocity. The presence of such singularities leads to a continuous spectrum of Alfvén-wave frequencies, with associated eigenfunctions exhibiting logarithmic behaviour at the singular point. Well-behaved solutions can be obtained only by integrating the singular modes over the entire spectrum. In this way, it is possible to obtain the asymptotic behaviour of non-collective oscillatory modes of the continuous spectrum, which present a (dissipationless) damping proportional to the inverse power of time (Barston 1964). Using an initial-value approach (Laplace transform), rather than a normal mode approach, it is possible to show that the solution also exhibits collective modes of oscillation (the surface eigenmodes) with position-independent frequency and exponential damping (resonant absorption) (Sedláček 1971). The damping rate found in this

† Permanent address: Scuola Normale Superiore, 56100 Pisa, Italy.



# THE PHYSICS OF THERMAL INSTABILITY IN TWO DIMENSIONS

L. SPARKS and G. VAN HOVEN

*Department of Physics, University of California, Irvine, CA 92717, U.S.A.*

(Received 10 January, 1985)

**Abstract.** Previous studies of a thermal (radiative) instability in a sheared magnetic field have shown that, under solar coronal conditions, cool condensations can form in a small neighborhood about the shear layer. Such results have served to model the formation of solar filaments (or prominences) observed to occur above photospheric magnetic polarity-inversion lines. A surprising conclusion of these studies is that the width of the condensation does not depend on the thermal conductivity ( $\kappa_{\parallel}$ ). By examining the mass-flow patterns of two-dimensional condensations in the absence of thermal conduction, we demonstrate that local plasma dynamics and the constraints imposed by boundary conditions are together sufficient to explain the size of the condensation width. In addition we present the results of a series of numerical calculations which illustrate the characteristic mode structure of sheared-field condensations.

## 1. Introduction

Solar prominences, interstellar clouds, and condensations in planetary nebulae are examples of astronomical objects which owe their existence to a condensation process of nongravitational origin. It has been proposed (Parker, 1953; Field, 1965) that such phenomena occur as a consequence of an instability in the thermal equilibrium of a diffuse medium. The condensation mechanism relies on optically thin radiation whose dependence on thermodynamic variables (e.g., density and temperature) is such that a cool, dense perturbation loses more energy through radiation than it gains through adiabatic and non-adiabatic heating processes and thermal conduction. As it cools, the pressure decreases, and the resulting mass inflow increases the density further.

Cool condensations in the solar corona (called filaments when viewed against the disk and prominences when viewed above the limb) are often observed to form above a magnetic neutral (polarity-inversion) line in regions of increasing magnetic field shear as indicated by photospheric magnetograms (Martin, 1973; Leroy, 1978). Local heat conduction, which is sharply collimated in a direction parallel to the magnetic field, would dominate radiation and other forms of energy transport if no field were present, thereby suppressing the thermal instability. Thus, one can expect the equilibrium field structure to exert a strong influence over the formation of prominences.

These empirical considerations have motivated previous computational studies of the dynamics of the thermal instability in a sheared magnetic field (Chiuderi and Van Hoven, 1979; Van Hoven and Mok, 1984). A surprising result of these studies is that the width of the condensation in a direction perpendicular to the shear layer does not depend on the thermal conductivity ( $\kappa_{\parallel}$ ) but only on the wavenumber and on the growth and radiation rates. Contrary to what might be expected, the condensation width does not correspond to those points at which the radiation loss is roughly balanced by parallel

## Resistive Alfvén normal modes in a non-uniform plasma

By G. EINAUDI† AND Y. MOK

Department of Physics, University of California, Irvine, California 92717

(Received 8 May 1985)

Resistive normal-mode solutions of the MHD equations are found numerically in a smooth, non-uniform, magnetic field. The  $(\alpha, S)$  boundary within which normal-mode solutions exist is explicitly computed, where  $\alpha$  is the normalized wavenumber and  $S$  the Lundquist number. As an extension of our previous analytic results (Mok & Einaudi 1985), the damping rate of these modes is computed to a higher accuracy, and is found to have an  $a + bS^{-1/2}$  dependence, where  $a$  and  $b$  are independent of  $S$ .

### 1. Introduction

The dissipation of Alfvén waves in a uniform plasma is generally believed to be weak. Recently, more attention has been turned to the damping of these waves in non-uniform configurations related to laboratory experiments and astrophysical conditions. The study of these waves has a wide range of applications. In particular, they may be responsible for the heating of solar coronal plasmas near active regions where the magnetic field is strongly inhomogeneous. Because they are easily generated in the low atmosphere by photospheric fluid motions, they are among the more plausible candidates for transporting energy to the corona. They may also be partly responsible for the heating of solar wind plasmas emerging from open magnetic regions where the field is far from uniform. In laboratory experiments, they are considered to be one of the potential candidates for heating fusion plasmas.

Studies of Alfvén waves in inhomogeneous media have been well documented (Hasegawa & Uberoi 1982, and references therein). In the ideal magneto-hydrodynamic (MHD) framework, these waves form a continuous spectrum, in contrast to the discrete normal modes which appear in the case of homogeneous media (Barston 1964; Sedlacek 1971). For the latter, small perturbations can oscillate at their local Alfvén frequencies and propagate at their local Alfvén velocities. Unfortunately, these normal-mode solutions usually possess logarithmic singularities in non-uniform media, which appear at the positions where their eigenfrequencies match the local Alfvén frequencies. In order to obtain a more nearly complete physical picture, it is necessary to take into account other phenomena, such as resistivity, viscosity or kinetic effects, which become important near the singularity of the solution.

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# VISCOUS NORMAL MODES ON CORONAL INHOMOGENEITIES AND THEIR ROLE AS A HEATING MECHANISM

R. S. STEINOLFSON<sup>1</sup>

Department of Physics, University of California, Irvine

AND

E. R. PRIEST, S. POEDTS, L. NOCERA, AND M. GOOSENS

Department of Applied Mathematics, The University, St. Andrews, Scotland

Received 1985 March 6; accepted 1985 October 30

## ABSTRACT

Viscous damping of Alfvén surface waves is examined both analytically and numerically using incompressible MHD. Normal modes are shown to exist on discontinuous as well as continuously varying interfaces in Alfvén speed. The waves experience negligible decay below the transition zone. High-frequency waves damp just above the transition region, while those of lower frequency lose energy further out. A comparison of dissipative decay rates shows that wave damping by viscosity proceeds approximately two orders of magnitude faster than by resistivity.

*Subject headings:* hydromagnetics — Sun: corona

## I. INTRODUCTION

The generation of magnetohydrodynamic (MHD) waves by photospheric and convection zone motions and the wave transfer of this mechanical energy into the corona continue to be viable mechanisms in some coronal heating theories (see, e.g., reviews by Hollweg 1983; Priest 1982; Kuperus, Ionson, and Spicer 1981; Wentzel 1981). An important aspect of the heating problem, which is receiving some attention currently, concerns the process by which the waves dissipate.

Waves may, of course, propagate throughout the corona, but those whose existence is intimately associated with inhomogeneities in the local Alfvén velocity appear to offer some heating advantages. Suggested processes by which the corona may absorb energy from these waves include the excitation of kinetic Alfvén waves (Hasegawa and Chen 1976), phase mixing (Pritchett and Dawson 1978; Heyvaerts and Priest 1983; Browning and Priest 1984), a Kolmogoroff turbulent cascade (Hollweg and Sterling 1984; Nocera and Priest 1984), and resistive normal-mode decay (Steinolfson 1985; Mok and Einaudi 1985). Resonant absorption of such waves has been considered by, for example, Ionson (1978), Pritchett and Canobbio (1981), Rae and Roberts (1981), and Sakurai and Granik (1984).

The propagation of surface waves as normal modes on magnetic or density discontinuities has been established using ideal MHD (e.g., Rae and Roberts 1981; Hasegawa and Uberoi 1982). When the discontinuity is replaced with a smooth profile, the ideal equations contain a singularity leading to the possibility of resonant absorption. A particularly lucid description of the singular problem is given by Lee and Roberts (1986). The presence of dissipation, however, removes the singularity. For instance, it has been shown that decaying normal-mode solutions occur on a smooth inhomogeneity when a finite resistivity is introduced (Steinolfson 1985).

The present paper considers the effect of viscosity on surface waves for both discontinuous and smooth variations in the Alfvén speed. The viscous tensor given by Braginskii (1965) is

used. A related problem was studied by Gordon and Hollweg (1983) for the discontinuous density case. They used the surface wave solution from ideal MHD and computed the potential heating rate due to wave fluctuations, while we self-consistently determine the viscous normal-mode behavior. In addition, their analysis only allowed velocity and magnetic field fluctuations perpendicular to the initial magnetic field. We include parallel fluctuations since it has been shown previously that they may be important in the dissipative process (Steinolfson 1984, 1985). Despite these differences in approach, the two studies are complementary with generally similar results and conclusions.

After establishing the existence of decaying viscous normal modes, we compare the viscous dissipation rate with that due to resistivity. The viscous decay rate is always about two orders of magnitude faster than the comparable resistive rate.

## II. LINEAR EQUATIONS

Single-fluid theory is used, and the corona is assumed to be incompressible and initially stationary. For a simplifying slab geometry, the ambient magnetic field is taken to coincide with the  $z$ -axis (perpendicular to the solar surface). Inhomogeneities, which in some cases we treat as a discontinuity, in either the magnetic field or density (i.e., the Alfvén speed) occur along the  $x$ -direction. Despite these variations in the initial quantities, we assume that the dissipation coefficients remain constant.

Since we are mainly interested in outward-propagating disturbances, the wave-propagation vector is required to be parallel to the magnetic field. A perpendicular component of wave propagation would not be expected to produce any new physical effects. For this assumed parallel orientation, there are no changes in the dependent variables along the  $y$ -axis.

Of the five ion viscosity coefficients given by Braginskii (1965), one ( $\nu_0$ ) is several orders of magnitude larger than the others for typical solar coronal conditions ( $\nu_0 = 0.61 \text{ g cm}^{-1} \text{ s}^{-1}$ ,  $\nu_1 = 2.8 \times 10^{-11} \text{ g cm}^{-1} \text{ s}^{-1}$ ,  $\nu_2 = 4\nu_1$ ,  $\nu_3 = 3.9 \times 10^{-6} \text{ g cm}^{-1} \text{ s}^{-1}$ ,  $\nu_4 = 2\nu_3$  for number density  $n = 10^9 \text{ cm}^{-3}$ , temperature  $T = 2 \times 10^6 \text{ K}$ , magnetic field  $B = 3.725 \text{ G}$ ). When the four smaller coefficients are neglected, a considerable sim-

<sup>1</sup> Present address: Department of Physics, University of Texas at Austin

## THE GROWTH OF RADIATIVE FILAMENTATION MODES IN SHEARED MAGNETIC FIELDS

Gerard Van Hoven  
Department of Physics  
University of California  
Irvine, CA 92717

### ABSTRACT

Observations of prominences show them to require well-developed magnetic shear and to have complex small-scale structure. We show here that these features are reflected in the results of the theory of radiative condensation. We have studied, in particular, the influence of the nominally negligible contributions of perpendicular (to  $\mathbf{B}$ ) thermal conduction. We find a large number of unstable modes, with closely spaced growth rates. Their scale widths across  $\mathbf{B}$  show a wide range of longitudinal and transverse sizes, ranging from much larger than to much smaller than the magnetic shear scale, the latter characterization applying particularly in the direction of shear variation.

### INTRODUCTION

Coronal prominences owe their existence to a condensation process which occurs as a consequence of an instability in the thermal equilibrium of a diffuse medium (Parker, 1953; Field, 1965; Hildner, 1974). The condensation mechanism relies on optically thin radiation whose dependence on thermodynamic variables (e.g., density and temperature) is such that a cool, dense perturbation loses more energy through radiation than it gains through adiabatic and non-adiabatic heating processes and thermal conduction.

Prominences and filaments (as seen on the disk) in the solar atmosphere are often observed to form above a magnetic neutral (polarity-inversion) line in regions of increasing magnetic field shear as indicated by photospheric magnetograms (Martin, 1973; Leroy, 1978). Local heat conduction, which is strongly attenuated in directions perpendicular to the magnetic field, would dominate radiation and other forms of energy transport if no field were present, thereby suppressing the thermal instability. Thus, one can expect the equilibrium field structure to exert a strong influence over the formation of prominences.

These empirical and physical considerations have motivated previous computational studies of the dynamics of the thermal instability in a sheared magnetic field (Chiuderi and Van Hoven, 1979; Van Hoven and Mok, 1984; Van Hoven et al., 1984; Sparks and Van Hoven, 1985). One result of these theoretical studies is that a sheared background field is necessary for the existence of a true localized

## SOLAR FLARE PRECURSORS

G. Van Hoven\* and G. J. Hurford\*\*

\*Department of Physics, University of California, Irvine, CA 92717,  
U.S.A.

\*\*Solar Astronomy, California Institute of Technology, Pasadena,  
CA 91125, U.S.A.

### ABSTRACT

We describe recent progress in the study of flare precursors and onset. We provide some new theoretical results on filament formation and eruption, including especially the dynamical effects of magnetic fields. We find ample evidence that energetic processes are already at work in the onset phase, a few minutes before the rapid rise of the hard-radiation impulse. In particular, the prevalence of soft and hard X-ray preheating (to  $< 10^6$  K) is detailed, along with a connection to the launch of coronal mass ejections. A possible interpretation of the empirical time profiles near onset is that "preheating" signifies that the flare has slowly started, and the transition to the impulsive stage then represents a change of phase in the flare-instability process. Finally, the problem of the interpretation of microwave preflare data is delineated. This represents the final report of this part of the Flare Build-up Study.

### INTRODUCTION

By the onset phase of a solar flare, we mean those phenomena that occur in the few minutes prior to the impulsive phase and have a direct, physical and perhaps irreversible connection to the subsequent rapid conversion of magnetic energy. The existence of phenomena on such a timescale, intermediate between the hours to days of active-region evolution and the seconds to tens of seconds of the impulsive phase, has been established beyond question over the last solar maximum. A wide variety of such manifestations has been observed, ranging from X-ray and UV emission, through filament activation and H $\alpha$  brightenings, to microwave polarization changes.

It might be argued that studying the onset of a flare is a waste of time, because, given the overstable magnetic field, something is bound to trigger it. However, the constrained rate of the initial energy growth, as contrasted with the rapid subsequent release, indicates that a different regime of plasma physics is relevant, which can be observed without distractions from the overwhelming manifestations of the main flare. Furthermore, a physical understanding of the onset phase is vital, if short-term flare prediction /1/ is to become a reality.

The extensive observations of onset-phase phenomena during the past solar maximum are a result of the application of the observing philosophy that lay at the heart of the Flare Buildup Study /2/; namely, one should observe active regions as continuously as possible until (not after) they flare. The preflare phase has therefore been the subject of study of the Flare Precursor and Onset (FPO) group of the Flare Buildup Study/Solar Maximum Analysis program. Preliminary results were reported previously /3/. This paper summarizes the deliberations of the FPO group at a meeting at Sacramento Peak Observatory in August 1985, and the observational and theoretical progress made in the last two years. For more complete reviews of earlier work, the reader is referred to the papers by Van Hoven /4/, Martin /5/ and Priest et al. /6/.

We will organize our discussion of flare indicators on a temporal basis by beginning with early filament evolution and ending with the late onset emission rise.

### FILAMENT FORMATION

One of the first manifestations of active-region development leading to a flare is an increase of magnetic shear and the subsequent condensation of a filament /7/. The cool filament appears above the polarity-inversion line, where the flare will later occur, and arises for a similar magnetic-topology reason /8, 3/.

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# IMPULSIVE PHASE SOLAR FLARE X-RAY POLARIMETRY

Gary Chanan

Department of Physics  
University of California  
Irvine, California 92717

A. Gordon Emslie\*

Department of Physics  
University of Alabama  
Huntsville, Alabama 35899

Robert Novick

Columbia Astrophysics Laboratory  
Columbia University  
New York, New York 10027

## ABSTRACT

The pioneering observational work in solar flare X-ray polarimetry was done in a series of satellite experiments by Tindo and his collaborators in the Soviet Union; initial results showed high levels of polarization in X-ray flares (up to 40%), although of rather low statistical significance, and these were generally interpreted as evidence for strong beaming of suprathermal electrons in the flare energy release process. However, the results of the polarimeter flown by the Columbia Astrophysics Laboratory as part of the STS-3 payload on the Space Shuttle by contrast showed very low levels of polarization. The largest value - observed during the impulsive phase of a single event - was  $3.4\% \pm 2.2\%$ . At the same time but independent of the observational work, Leach and Petrosian (1983) showed that the high levels of polarization in the Tindo results were difficult to understand theoretically, since the electron beam is isotropized on an energy loss timescale - an effect which substantially reduces the expected levels of polarization, although not to zero. A subsequent comparison by Leach, Emslie, and Petrosian (1985) of the impulsive phase STS-3 result and the above theoretical treatment shows that the former is consistent with several current models and that a factor of  $\sim 3$  improvement in sensitivity is needed to distinguish properly among the possibilities. In addition, there is reason to expect stronger polarization effects at higher energies: There may have been a strong thermal component to the flare at the energies seen by the STS-3 instrument (predominantly below 10 keV), and in addition the preponderance of  $\gamma$ -ray ( $\geq 300$  keV) events on the solar limb (Rieger et al. 1983) suggests that beaming must be important at sufficiently high energies.

\* Presidential Young Investigator

# NONLINEAR RADIATIVE CONDENSATION IN A SHEARED MAGNETIC FIELD

G. VAN HOVEN AND L. SPARKS

Department of Physics, University of California, Irvine

AND

D. D. SCHNACK

Science Applications International Corporation, San Diego

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## ABSTRACT

The thermal instability, in which plasma cooling and condensation are driven by increasing radiation losses, is believed to underlie a number of filamentation processes in astrophysics. Most often, however, the local region must be threaded by a sheared magnetic field, so that the strong temperature-equalizing influence of electron thermal conduction can be suppressed. This fact, which is well known from the solar example, leads to difficulties in treating the dynamics that accompany the filamentation energetics. Previous linear calculations have delineated this radiative instability and have shown the influence of magnetically channeled heat and mass flows. This *Letter* describes a unique, well-resolved, two-dimensional, nonlinear, numerical simulation of the formation of a filament in a force-free field. The condensation is initiated by a linearly unstable mode and widens until it is slowed by parallel (to  $B$ ) thermal conduction. During the nonlinear evolution, the minimum temperature falls from  $10^6$  K to  $10^4$  K and eventually reaches a state of local thermal equilibrium in about five e-folding times.

*Subject headings:* hydromagnetics — magnetic fields — plasmas — radiation mechanisms — Sun: prominences

## 1. INTRODUCTION

Nongravitational condensations, such as solar prominences, owe their existence to the fact that thermal equilibrium in a plasma can be unstable in the presence of a radiation function that decreases strongly with temperature (Parker 1953). The first complete dynamic treatment of this thermal instability in a uniform magnetized medium was given by Field (1965) who detailed the linear growth under various conditions. Nonlinear dynamic computations of radiative condensation in a uniform magnetic field have been attempted by Raju (1968), using a one-dimensional model, and by Hildner (1974) in two dimensions, although he was not able to carry his computations to local thermal equilibrium because of boundary interactions.

It is a crucial empirical fact, however, that filaments form in *nonuniform* regions of magnetic polarity inversion (Martin 1973). Such configurations serve to suppress the thermal conduction, which normally (in a high-temperature field-free region) provides a larger energy-flux contribution than radiation, by increasing the parallel (to  $B$ ) distance to the boundaries. Chiuderi and Van Hoven (1979) analyzed filament condensation in a sheared field and produced the first localized two-dimensional unstable mode. This effort was extended by Van Hoven and Mok (1984) and Van Hoven, Sparks, and Tachi (1986) to include the essential effects of perpendicular thermal conduction.

In this *Letter*, we report the first results of a two-and-one-half-dimensional (vector fields have components in the ignorable direction) nonlinear simulation of unstable radiative condensation in a sheared, force-free, magnetic field, for

conditions applicable to the solar corona. The treatment includes a reasonably complete energy-transport model and is therefore quite general and relevant to a number of astrophysical situations, as was emphasized by Field (1965).

## II. PHYSICAL MODEL

The situation of interest in the thermal instability can be described by the equations of *ideal* (infinite electrical conductivity) magnetohydrodynamics (Field 1965; Hildner 1974), relating the plasma pressure  $p$ , density  $\rho$ , velocity  $\mathbf{v}$ , and magnetic field  $\mathbf{B}$ . To these are added the ideal gas law and a suitable form for the energy equation

$$\frac{dp}{dt} = -\gamma p \nabla \cdot \mathbf{v} + (\gamma - 1) [H_0 - \rho^2 \Phi(T) + \nabla \cdot \mathbf{\kappa} \cdot \nabla T], \quad (1)$$

where  $H_0$  is an equilibrium heating function (which, in the absence of an accepted parametric model, is chosen to balance the radiative losses at  $t = 0$ ),  $\Phi(T)$  specifies the temperature variation of the radiation loss, and  $\mathbf{\kappa}$  is the anisotropic thermal conductivity which satisfies  $\kappa_{\perp} \ll \kappa_{\parallel}$  (Spitzer 1962). The  $T$ -dependence of the radiation law, due to optically thin emission from variably ionized trace constituents, can be approximated by expressions of the form  $\Phi(T) = RT^r$ , where  $R$  and  $r$  are piecewise constant ( $r = 0.5$ ,  $T > 10^7$  K;  $r = -1.0$ ,  $10^{3.9} < T < 10^7$ ;  $r = -2.5$ ,  $10^{3.5} < T < 10^{3.9}$ ;  $r = 0$ ,  $10^{4.9} < T < 10^{3.5}$ ;  $r = 1.8$ ,  $10^{4.2} < T < 10^{4.9}$ ; and  $r = 7.4$ ,  $T < 10^{4.2}$  [Hildner 1974]). The global equilibrium magnetic

## ALFVÉN WAVE DISSIPATION IN THE SOLAR ATMOSPHERE<sup>1</sup>

G. EINAUDI

Scuola Normale Superiore, Pisa

AND

YUNG MOK

Department of Physics, University of California, Irvine

Received 1986 August 27; accepted 1987 January 22

### ABSTRACT

Dissipative Alfvén waves in a nonuniform plasma are studied by using a normal mode analysis. Specific magnetic geometries are used to simulate various magnetic structures in the solar atmosphere. We have computed the real and imaginary parts of the eigenfrequency for each of the configurations and found that these eigenmodes can deposit a substantial amount of energy to the corona because of their short damping distances.

*Subject headings:* plasmas — Sun: corona

### 1. INTRODUCTION

The possibility of heating the solar corona by the dissipation of Alfvén waves has been explored by many authors. A number of aspects of such theories have been discussed in various review articles (e.g., Kuperus, Ionson, and Spicer 1981). A wave theory for coronal heating must answer two fundamental questions. The first one is whether the energy requirement of the corona can be met by the observed wave amplitudes. The second one concerns the capability of the waves to dissipate a considerable part of their energy at the coronal level.

As far as the first point is concerned, there is ample observational evidence indicating the existence of nonthermal motions in the chromosphere and coronal base. These motions, which are unresolved in space and time, are likely to be fluid motions or propagating hydromagnetic waves, or both. Although there are constraints on the amount of energy being carried by these waves, nonthermal motions of the order of  $10\text{--}30\text{ km s}^{-1}$  have been observed (e.g., Bonnet 1978; Cheng, Doschek, and Feldman 1979; Doschek and Feldman 1977; Feldman, Doschek, and Tousey 1975). If these observed motions are present in a large volume of plasma at the coronal base, it seems that they can provide sufficient energy to heat the corona (Hollweg 1983).

The great difficulty has been considered to be the low efficiency of viscous or ohmic dissipation, which is very weak in a uniform plasma and therefore unable to deposit a significant amount of energy within the distance scale of the corona. However, since the corona is far from being a uniform medium, it is important to know the properties of propagation and dissipation of Alfvén waves in nonhomogeneous plasmas in order to properly understand the role of the waves in coronal heating.

It is well known that no dispersion relation analogous to the uniform case can be found for a nonuniform plasma (e.g., Barston 1964; Sedlacek 1971) if the effects of a dissipation mechanism are neglected. This is due to the singular behavior of the solutions of the ideal hydromagnetic equations in the neighborhood of resonant surfaces, where the phase velocity of the mode is equal to the local Alfvén velocity. Modes with real frequencies, as in the framework of the ideal MHD equations (Lee 1980), exhibit a logarithmic singularity at this surface (Goedbloed 1983), making the number of boundary conditions to be imposed insufficient to determine the eigenvalue, thus indicating a continuous spectrum. The difficulty arising from this ideal MHD singularity can be circumvented by simply introducing a complex eigenfrequency, leading to a discrete spectrum of damped regular modes (Tataronis and Grossman 1973). As pointed out by Lee and Roberts (1986), the decay rate of these modes must not be interpreted as a dissipation rate, because of the absence of true dissipation in the ideal MHD framework, but rather as a mode conversion of the collective disturbances into local oscillations within a layer around the resonant surface. One of the effects of the local oscillations is to increase the spatial gradients of the wave excitations, making it necessary to consider dissipative processes within the resonant layer.

Some of the mathematical properties of the equations with resistivity added as a dissipation mechanism have been studied by several authors (Riedel 1986; Sakurai and Granik 1985; Pao and Kerner 1985; Dewar and Davies 1984; Heyvaerts and Priest 1983; Kappraff and Tataronis 1977). An explicit solution has been obtained by Mok and Einaudi (1985, hereafter Paper I), by introducing the electron resistivity or ion viscosity, or both, as true dissipation mechanisms. This solution has a mode-damping rate which is independent of the magnitude of the dissipation coefficients in first approximation. Interestingly, the damping rate is the same as the mode-conversion rate found in the ideal MHD framework, meaning that all the energy of the collective disturbances which is converted into local oscillations is dissipated. The property of the dissipative layer to be a perfect absorber of Alfvén waves without reflection occurring has been shown by Bertin, Einaudi, and Pegoraro (1986). As a consequence of this local behavior of the solution around the resonant surface, the eigenfrequency of the mode depends only on the equilibrium parameters and on its wavelength, but not the detail of the dissipation mechanism.

In this paper we consider several magnetic profiles that model different coronal regions and compute the real frequency as well as the damping rate of the Alfvén waves traveling along the magnetic field. The possible rate of energy deposition by the waves into the

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# Radiative instabilities in a sheared magnetic field

J. F. Drake

*Laboratory for Plasma and Fusion Energy Studies, University of Maryland, College Park, Maryland 20742*

L. Sparks and G. Van Hoven

*Department of Physics, University of California, Irvine, California 92717*

(Received 3 August 1987; accepted 29 December 1987)

The structure and growth rate of the radiative instability in a sheared magnetic field  $B$  have been calculated analytically using the Braginskii fluid equations. In a shear layer, temperature and density perturbations are linked by the propagation of sound waves parallel to the local magnetic field. As a consequence, density clumping or condensation plays an important role in driving the instability. Parallel thermal conduction localizes the mode to a narrow layer where  $k_{\parallel} = k \cdot B/|B|$  is small and stabilizes short wavelengths  $k > k_c$ , where  $k_c$  depends on the local radiation and conduction rates. Thermal coupling to ions also limits the width of the unstable spectrum. It is shown that a broad spectrum of modes is typically unstable in tokamak edge plasmas and it is argued that this instability is sufficiently robust to drive the large-amplitude density fluctuations often measured there.

## I. INTRODUCTION

Radiative cooling plays a fundamental role in the formation of a number of astronomical objects, including solar prominences, interstellar clouds, and planetary nebulas.<sup>1,2</sup> The literature on solar prominences is particularly extensive.<sup>3</sup> Prominences are regions of high density and low temperature that form in the solar corona. They can be up to a hundred times denser and a hundred times cooler than the ambient background with scale sizes of  $10^4$ – $10^5$  km.

More recently, the importance of radiative cooling in the edge region of laboratory tokamak discharges has been recognized. The onset of high density disruptions, which limit the maximum density that can be confined in a discharge, is triggered by impurity radiation in the edge plasma.<sup>4–9</sup> A new phenomenon, the marfe, has also been documented in a variety of tokamaks.<sup>10–14</sup> Somewhat below the density limit, a toroidally symmetric region of cold, high density plasma forms in the edge region on the small major radius side of the plasma. The peak density in the marfe can become comparable to that in the center of the main body of the plasma.<sup>14</sup> It has been proposed that the marfe, like the solar prominence, arises from a radiative condensation process.<sup>15–17</sup>

Short scale length fluctuations in the edge density of tokamak plasmas have been measured for more than a decade.<sup>18–22</sup> The spectrum of these fluctuations is largely two-dimensional, perpendicular to the local magnetic field. Unlike the marfe, they therefore track the local magnetic field. The level of fluctuations can be substantial, of the order of the background density ( $\bar{n}/n \sim 1$ ).

It has been proposed that rippling modes, driven by the local gradient in parallel resistivity, are the source of these fluctuations.<sup>23</sup> However, low current in the plasma edge combined with diamagnetic propagation and parallel thermal conduction tends to weaken the rippling mode.<sup>24</sup> Therefore it does not appear that this instability is sufficiently robust to drive fluctuations to the high levels often seen experimentally.

Finally, there is recent experimental evidence that the amplitude of the fluctuations increases rapidly as the marfe density threshold is approached. We propose that the short-wavelength edge-density turbulence is also driven by the thermal condensation instability.

The most primitive radiative thermal instability arises from the amplification of a simple flute ( $B \cdot \nabla = 0$ ) perturbation of the local temperature. The growth rate is given by<sup>1</sup>

$$\gamma = -\frac{1}{3\pi} \frac{\partial L}{\partial T}, \quad (1)$$

where  $L(n, T) = \pi^2 g(T)$  is the radiation rate. If a drop in the local temperature  $T$  increases the radiation rate ( $\partial L / \partial T < 0$ ), the temperature will fall even farther so that the system is unstable. When the parallel wave vector  $k_{\parallel}$  is non-zero the sound wave can flatten the pressure along the magnetic field line and perturbations of the temperature and density are coupled. The local growth rate of the thermal condensation instability is then given by

$$\gamma = \frac{1}{5\pi} \left( \frac{2L}{T} - \frac{\partial L}{\partial T} - k_{\parallel}^2 \kappa_{\parallel e} \right), \quad (2)$$

where  $\kappa_{\parallel e}$  is the coefficient of electron thermal conduction. The first term in Eq. (2) is always destabilizing and it arises from the density perturbation. Because of parallel pressure balance, a local decrease in the temperature causes an increase in the local density and thus an increase in the radiation rate. The growth of perturbations arising from the increase in the density (in contrast to the decrease in the temperature alone) is referred to as the thermal condensation instability. Parallel thermal conduction is stabilizing. In an initially homogeneous plasma in a uniform magnetic field, the parallel wave vector can be made arbitrarily small and instability is always possible as long as  $\partial L / \partial T < 2L / T$ .

In configurations of interest in both laboratory and astrophysical applications, however, the geometry of the system is often not so simple. There is observational evidence

## AN MHD SIMULATION OF PLASMA FLOW PAST IO: ALFVÉN AND SLOW MODE PERTURBATIONS

 Jon A. Linker<sup>1,2</sup>, Margaret G. Kivelson<sup>1</sup> and Raymond J. Walker

Institute of Geophysics and Planetary Physics, University of California, Los Angeles

**Abstract.** We have studied the flow of plasma past Io using a time dependent, three-dimensional magnetohydrodynamic (MHD) simulation. In addition to observing the "Alfvén wing", a standing Alfvén wave perturbation expected from analytic theory, we have found that the other MHD modes contribute important perturbations. In particular, standing slow mode perturbations also are present in the flow.

## Introduction

The interaction of Io, Jupiter's innermost Galilean satellite, with the Io torus plasma, has long been a subject of interest. Much of the discussion has been formulated in terms of perturbations carried by Alfvén waves. It was Drell et al. [1965], in studying drag effects on satellites in the earth's ionosphere, who first demonstrated that the presence of a conductor in a flowing plasma results in a standing Alfvén perturbation in the rest frame of the conductor. An Alfvén wave is launched when a conducting obstacle is placed in motion relative to a magnetized fluid. For the Alfvén wave in a uniform plasma, the group velocity is along or opposite to the magnetic field in the plasma rest frame. In the conductor's rest frame, the Alfvén perturbation is also carried downstream by the background flow. The leading edge of the perturbation region occurs along the Alfvén characteristic, at an angle  $\tan^{-1} M_A$  to the background magnetic field, where  $M_A$  is the Alfvén Mach number. This region, where the perturbation currents flow, is known as the "Alfvén wing".

Subsequent work has applied the concept of an Alfvén wave interaction to the flow of the torus plasma past Io [e.g., Goertz, 1980; Neubauer, 1980; Southwood et al., 1980; Wright and Southwood, 1987]. Although these papers give a good description of the Alfvén perturbations far from Io, they consider only the Alfvén mode and do not model perturbations in the flow close to Io. Wolf-Gladrow et al. [1987] modeled numerically the magnetic field and current perturbations close to Io. However, they neglected the slow mode and other finite plasma beta effects as well.

In this paper we present results from a three-dimensional MHD simulation of the flow of the torus plasma past Io. MHD simulations have been used previously to model the interaction of the solar wind with the earth's magnetosphere [e.g., Leboeuf et al., 1981; Wu et al., 1981; Brecht et al., 1982; Ogino, 1986; Fedder and Lyon, 1987] and with

comets [e.g., Schmidt and Wegmann, 1982; Ogino et al., 1988]. In our simulations we solve the time-dependent, compressible MHD equations, so that all of the MHD wave modes are included. We have found that in addition to the Alfvén mode, other MHD modes can contribute important perturbations. In this paper we describe the slow mode perturbations caused by the interaction of Io with the torus plasma.

## Description of the Simulation

The details of the numerical techniques used in our simulation are discussed by Linker [1987] and will be presented in a forthcoming paper; here we briefly outline the simulation model. We solve the following form of the normalized MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{v}) = 0 \quad (1)$$

$$\frac{d}{dt} (P \rho^{-\gamma}) = \frac{\gamma-1}{\rho^{\gamma}} \left( R J^2 - \frac{1}{R_e} \mathbf{W} : \nabla \bar{v} \right) \quad (2)$$

$$\begin{aligned} \frac{\partial}{\partial t} (\rho \bar{v}) + \nabla \cdot (\rho \bar{v} \bar{v} + \left( P + \frac{B^2}{2} \right) \mathbf{I} - \bar{B} \bar{B}) \\ = -\frac{1}{R_e} \nabla \cdot \mathbf{W} \end{aligned} \quad (3)$$

$$\frac{\partial \bar{B}}{\partial t} - \nabla \times (\bar{v} \times \bar{B}) = -\nabla \times (R \bar{J}) \quad (4)$$

In the above equations,  $\rho$  is the density,  $\bar{v}$  is the velocity,  $P$  the plasma pressure,  $\bar{B}$  the magnetic field, and  $\bar{J} = \nabla \times \bar{B}$  is the current density.  $\mathbf{W}$  is a viscous stress tensor used for numerical stability purposes.  $R_e$  is the fluid Reynolds number,  $R$  is the resistivity, and  $\gamma = 5/3$ . For the results shown in this paper,  $R_e = 50$  and  $R_m$ , the magnetic Reynolds number ( $\frac{V_A L}{R}$ , where  $V_A$  is the Alfvén speed and the length scale  $L$  is an Io diameter) is 100. The dissipative terms are included to help damp out short wavelength ripples generated by numerical dispersion, while leaving the longer wavelength phenomena minimally affected. We have performed simulations with magnetic and fluid Reynolds numbers in the range of 10-200. The results remain qualitatively the same in this range, but higher levels of numerical fluctuations occurred in the higher Reynolds number cases.

The equations are solved as an initial value problem in spherical coordinates by using a two-step Lax-Wendroff finite difference scheme. There are two boundaries in the simulation: at Io's surface and at the outer boundary of the simulation region ( $r = 10$  Io radii from the center of Io). Figure 1 shows the coordinate system for the simulation. Io is centered at  $r = 0$ , the center of the coordinate system. In our simulation, we assume that Io has no intrinsic magnetic field. If this assumption is correct, in

<sup>1</sup>Also at Department of Earth and Space Sciences, University of California, Los Angeles.

<sup>2</sup>Now at Physics Department, University of California, Irvine.

## Spectral analysis of turbulent effects on resistivity and the tearing instability

By D. DEEDS AND G. VAN HOVEN

Department of Physics, University of California, Irvine, U.S.A.

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Biskamp and Welter (1983) have defined an anomalous resistivity due to short-wavelength turbulence. They reported that this resistivity can be of either sign, and that negative anomalous resistivity in particular can affect the growth of the tearing instability. We use a spectral numerical-simulation code and ancillary diagnostics to analyse the behaviour of resistive magnetic tearing in the presence of turbulence of the sort postulated by Biskamp and Welter. We find that, in general, the 'anomalous resistivity' tends to return quickly towards zero even when artificially supported away from zero, and that its effect on tearing-mode behaviour is not consistent with its interpretation as a resistivity. We investigate analytically the behaviour reported by Biskamp and Welter, and the behaviour we observe. We also argue that, while not meaningful as a true resistivity, the 'anomalous-resistivity' parameter is a useful diagnostic showing the energy balance of the system - a property we refer to as Alfvénicity - illustrating, for example, the onset of nonlinearity in the tearing process.

### 1. Introduction

Magnetic reconnection, or tearing, is believed to be an important - perhaps the fundamental - mechanism for the release of magnetic energy to other forms of energy in both astronomical and laboratory plasmas (Priest 1985). A number of hydromagnetic simulations have been performed to investigate the linear and nonlinear dissipative evolution of reconnection under various initial conditions and other assumptions (Furth, Killeen & Rosenbluth 1963; Van Hoven & Cross 1973; Matthaeus & Montgomery 1981). The essence of magnetic tearing is the resistive diffusion of a sheared field toward a more energetically favourable configuration, allowing the conversion of magnetic energy to bulk motion and heat.

Of significant interest is the efficiency of tearing: how fast will it proceed, and how completely? This will certainly depend on the various parameters and conditions used: a major objective of research into tearing has been to achieve theoretical rates and yields consistent with (primarily) solar-activity observations (Van Hoven & Cross 1973). That there is no simple connection between the energetic requirement of magnetic release and the empirical output levels suggests the complexity of the processes involved.

One concept that has been explored in the search for faster tearing is turbulence (Biskamp & Welter 1983; Matthaeus & Lamkin 1986). The premise here is that short-wavelength hydromagnetic excitations couple nonlinearly

# PROSPECTS FOR SOLAR FLARE X-RAY POLARIMETRY

GARY CHANAN

*Department of Physics, University of California, Irvine, CA 92717, U.S.A.*

A. GORDON EMSLIE\*

*Department of Physics, University of Alabama, Huntsville, AL 35899, U.S.A.*

and

ROBERT NOVICK

*Columbia Astrophysics Laboratory, Columbia University, New York, NY 10027, U.S.A.*

(Received 23 December, 1985)

**Abstract.** The history of solar flare X-ray polarimetry is reviewed and it is shown that as yet, there is no experimental evidence for such polarization. The present experimental limits are at the level of a few percent but these results may be biased by a large thermal component at low energies which may decrease the apparent polarization. To avoid this difficulty it will be necessary to make observations at higher energies where thermal emission is less important.

The theoretical estimates of the polarization expected in the solar flare are also reviewed. The best present theoretical estimates are in the range of a few percent and are consistent with the present experimental limits.

In this paper we discuss a new satellite instrument that has sufficient sensitivity at high energies to detect the polarization that is predicted by the present theories. The instrument sensitivity for a moderate (M class) event approaches polarization levels of 1% in each of 7 energy bins spanning the 10 to 100 keV range for integration times as short as 10 s. Comparable results can be obtained for an X class flare in 1 s.

## 1. Solar Flare X-Ray Polarimetry

The idea that X-ray emission from solar flares might be linearly polarized and that polarization measurements could, therefore, provide a strong flare diagnostic was first discussed by Korchak (1967) and Elwert (1968). Subsequent theoretical investigations (Elwert and Haug, 1970, 1971; Haug, 1972; Brown, 1972; Henoux, 1975; Langer and Petrosian, 1977; Bai and Ramaty, 1978; Emslie and Brown, 1980) have resulted in polarization predictions for a variety of models. There are two extreme classes of models under investigation, termed 'thermal' and 'non-thermal', whose physical difference lies principally in whether the electrons responsible for the bremsstrahlung are part of a relaxed distribution or of a suprathermal tail. Although some form of hybrid model (e.g., Emslie and Vlahos, 1980) is probably appropriate for actual events, the basic components differ significantly in their polarization predictions: the thermal models predict polarizations of at most a few percent, due to either photospheric back-scatter of primary photons (Henoux, 1975), or an anisotropy in the source electron velocity distribution, caused by the presence of a field-aligned thermal conductive flux (Emslie and Brown, 1980). The beamed or linear bremsstrahlung models, on the other hand,

\* Presidential Young Investigator.

## CREATION OF CURRENT FILAMENTS IN THE SOLAR CORONA

Z. MIKIĆ AND D. D. SCHNACK

Science Applications International Corporation, San Diego

AND

G. VAN HOVEN

Department of Physics, University of California, Irvine

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### ABSTRACT

It has been suggested that the solar corona is heated by the dissipation of electric currents. The low value of the resistivity requires the magnetic field to have structure at very small length scales if this mechanism is to work. In this paper we demonstrate that the coronal magnetic field acquires small-scale structure through the braiding produced by smooth, randomly phased, photospheric flows. The current density develops a filamentary structure and grows exponentially in time. Nonlinear processes in the ideal magnetohydrodynamic (MHD) equations produce a cascade effect, in which the structure introduced by the flow at large length scales is transferred to smaller scales. If this process continues down to the resistive dissipation length scale, it would provide an effective mechanism for coronal heating.

*Subject headings:* hydromagnetics — Sun: corona

### 1. INTRODUCTION

The physical processes which heat the solar and other stellar coronae are not well understood (Priest 1982, pp. 206–234). However, it is widely accepted that the magnetic field in the solar atmosphere plays a prominent role in the energy-deposition process. Based on this concept, it has been suggested that the corona is heated through the dissipation of electric currents. In one version of this picture, the shearing and compression of a large-scale field whose footpoints are anchored below the surface can occur as a result of convective motions in the photosphere. This deformation induces electric currents to flow in the corona. The length scale on which the footpoints are shuffled by the convective motions corresponds to the granulation scale length ( $l_g \sim 10^6$  m). If the coronal heating rate is to be accounted for by Joule dissipation of electric currents, the magnetic field is required to have structure at scale lengths several orders of magnitude smaller than  $l_g$ , due to the large value of the conductivity in the high-temperature corona. Thus, in order to explain coronal heating by the above mechanism, it is required to demonstrate how the magnetic field can acquire structure at length scales  $l \ll l_g$ .

The proposed explanation of the mechanism by which the magnetic field acquires small-scale structure is somewhat controversial. Parker (1972, 1983a, 1986) has noted from a perturbation analysis that, in general, footpoint displacements lead to a loss of equilibrium, inducing current sheets to form in the corona. He has argued that the equilibrium MHD equations require the small-scale field to be invariant along the direction of the large-scale field. In situations in which the footpoint motions produce braiding of the magnetic field (which are expected to typify quasi-random photospheric flows), it is not possible to meet this requirement. According to Parker, the field would then necessarily require the presence of current sheets. The rapid resistive reconnection of magnetic field at these sheets would provide enhanced dissipation over that produced by normal Joule dissipation of the large-scale field, and would allow the field to reach equilibrium with a much simpler topology. This process has been called "topological

dissipation" (Parker 1972). In Parker's hypothesis, then, the structure at  $l \ll l_g$  in the corona is formed trivially through the creation of current sheets.

In contrast, Rosner and Knobloch (1982) have noted that "the finite-amplitude behavior of a nonlinear system can be quite different from that predicted on the basis of small-parameter expansions." Van Ballegoijen (1985, 1988a), Antiochos (1987), and Zweibel and Li (1987) have noted that well-behaved, continuous photospheric flows produce coronal fields which are free of discontinuities in the absence of initial neutral points in the field. However, van Ballegoijen (1985) hypothesized that random smooth flows (which are characteristic of the flows produced by subphotospheric convection) produce structure in the magnetic field at arbitrarily small length scales in a "cascade" process. In a statistical analysis van Ballegoijen (1986) determined the rate at which the structure at  $l_g$  is expected to cascade to short length scales. For an idealized random flow profile van Ballegoijen (1988a) conjectured that the electric current density in the corona ought to build up exponentially in time. Indeed, a realization of the first few steps of this process (van Ballegoijen 1988b), involving the numerical solution of the *equilibrium* MHD equations, indicates that such flows rapidly introduce fine structure in the coronal field.

In this paper we describe dynamical calculations which address the issue of the creation of small-scale structure in magnetic fields. We solve the time-dependent three-dimensional ideal MHD equations in a geometry which is an idealization (Parker 1972) of the solar corona. The MHD equations are solved in the limit of vanishing gas pressure. We find that a sequence of smooth, randomly phased flows (like those used by van Ballegoijen 1988a, b) generates structure in the field at increasingly shorter lengths. This structure is evidenced by an exponentially growing current density (with step number in the flow sequence) and a cascade of spatial structure to smaller scales. The rate of generation of the electric current density is consistent with van Ballegoijen's (1988a) predictions. These simulations are dynamical generalizations of van

## SOLAR CORONAL LOOP HEATING BY CROSS-FIELD WAVE TRANSPORT

PETER AMENDT AND GREGORY BENFORD

Department of Physics, University of California, Irvine

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### ABSTRACT

Solar coronal arches heated by turbulent ion-cyclotron waves may suffer significant cross-field transport by these waves. Nonlinear processes fix the wave-propagation speed at about a tenth of the ion thermal velocity, which seems sufficient to spread heat from a central core into a large cool surrounding cocoon. Waves heat cocoon ions both through classical ion-electron collisions and by turbulent stochastic ion motions. Plausible cocoon sizes set by wave damping are in roughly kilometers, although the wave-emitting core may be only 100 m wide. Detailed study of nonlinear stabilization and energy-deposition rates predicts that nearby regions can heat to values intermediate between the roughly electron volt foot-point temperatures and the  $\sim 100$  eV core, which is heated by anomalous Ohmic losses. A volume of 100 times the core volume may be affected. This qualitative result may solve a persistent problem with current-driven coronal heating; that it affects only small volumes and provides no way to produce the extended warm structures perceptible to existing instruments.

*Subject headings:* hydromagnetics — Sun: corona — wave motions

### 1. INTRODUCTION

Coronal heating models often invoke anomalous Joule heating by current-driven instabilities (Tucker 1973; Heyvaerts 1974; Vlahos 1979; Hinata 1979). Of late, the electrostatic ion-cyclotron mode has emerged as a prime candidate for such a role, since it can be excited by lower electron-drift velocities and heats ions preferentially in the plane transverse to the magnetic field, lessening convective losses (Papadopoulos 1977).

The electrostatic ion-cyclotron mode is particularly suited for cross-field transport because it results from a trade-off between harnessing the electron drift energy along the magnetic field, versus the ion damping at the cyclotron frequency,  $\Omega_i$ . Thus the waves store most of their energy transverse to  $B(k_1 \gg k_2)$  and have frequency  $\omega \approx 1.1\Omega_i$ , just far enough from  $\Omega_i$  to avoid ion damping. The resonance broadening model seems to describe at least qualitatively the nonlinear dynamics (Benford 1976) in which the linear dispersion relation still describes the observed wave properties.

Benford (1983, hereafter Paper I) presented a detailed calculation for a coronal loop heated by small-scale current-carrying filaments ( $\sim 100$  m in radius). He found that only the ion cyclotron mode appeared, and the system quickly settled down to a quiescent regime in which convective losses caused the instability to flicker on and off, "percolating" the system to a steady temperature of  $\sim 100$  eV.

Benford's calculation neglected transverse (to  $B$ ) wave transport from the ion cyclotron modes, however. Here we make some estimates of the importance of this effect and argue that large annular regions transverse to a current-carrying zone may be heated by the instability. We describe in detail the process of nonlinear wave transport and saturation in the surrounding cocoon of initially cooler, roughly electron volt plasma, which is then warmed to intermediate temperatures. This effect may be important in describing how coronal energy moves from the hypothesized small current-carrying zones.

Although theory has found plausible ways to heat small regions to high temperatures, conventional thermal-

conduction mechanisms do not allow cross-field transport to large lateral volumes. Here we attempt to correct this deficiency, detailing the underlying plasma processes needed and exploring their ramifications. Once radiated from the small  $\sim 100$  m core, a spaghetti-like current-carrying strand, the ion cyclotron waves are slowly damped in the surrounding cocoon, heating it. Damping by ion-electron collisions sets a limit on the cocoon size,  $R$ , exceeding 1 km. Moreover, stochastic ion motions in the wave fields can heat ions and electrons. This constraint leads to  $R$  comparable to 1 kilometer for plausible turbulence levels. Thus sizable cocoons are unavoidable. The true value of  $R$  must take into account convective cooling along the field lines as well.

We consider these matters qualitatively, although with some quantitative treatment of energy transfer rates in the nonlinear stages. The aim of our work is to argue that such effects generally make lateral transport an important mechanism. Other schemes, such as electron cyclotron emission are also possible (Melrose and Dulk 1984).

We depend on the general approach of Paper I, adding in § II of this paper a qualitative sketch of how important and widespread the effect can be. Basically, we show how transverse wave loss sets an upper limit on the size,  $R$ , of any heated cocoon. This size is much larger than, say, the ion gyroradius because ordinary Coulomb processes are ineffective for solar coronal conditions. Section III explores in detail the underlying microscopic behavior of the ion cyclotron instability which supports the transverse energy transport. We summarize our conclusions in § IV.

### II. SIZE OF THE LUKEWARM COCOON

Consider a cylinder representing a straightened magnetic loop (Fig. 1). Currents flowing with drift speed  $u$ , exceeding the critical onset drift for ion cyclotron instability  $u_c$ , heat a core to temperature  $T_c$ , as described in Paper I. The system quickly reaches a "percolating" steady state characterized by marginal stability of the ion cyclotron modes. Convection down the field lines ( $z$ -axis) offsets wave deposition of energy into both ions

# Energetics and Dynamics of Solar Activity

Gerard Van Hoven

Department of Physics, University of California, Irvine

The activity of the Sun occurs in the solar atmosphere and is driven and confined by the Sun's magnetic field. The plasma atmosphere comprises the cool ( $\leq 10^4$  K) and dense chromosphere, an intermediate transition region, and the hot ( $T_e \geq 10^6$  K) and diffuse ( $n_e \geq 10^8$  cm $^{-3}$ ) corona; the solar atmosphere has been well observed from the ultraviolet through hard X rays, by the Skylab and Solar Maximum Mission spacecraft, among others. The atmospheric magnetic field has its source in the solar interior and is driven and energized by the global differential rotation and local turbulent motions at the visible photospheric surface; this field is mostly known from Zeeman-effect and microwave measurements. The magnetic field lines return to the surface in active regions and are open in coronal holes that provide the source for the solar wind.

The primary concern of the Solar Plasma Theory Group at the University of California, Irvine, involves the dynamics and energetics of magnetic activity. The coronal field is strong ( $B^2/2\mu_0 P \gg 1$ ), nonuniform, and stressed by currents driven from the surface. This field provides a source of stored energy and an anisotropic medium for the channeling of mass and energy flows in the atmospheric plasma.

There are a number of unsolved problems connected with observed solar activity. They are briefly stated (to be amplified in the following sections) here. How is the corona heated; that is, how is the necessary energy transported up the adverse temperature gradient from the photospheric source surface and deposited in the corona, apparently most effectively in strong-field regions? How does the atmospheric magnetic field channel and confine the ambient heat and mass flows so that the former are suppressed, allowing runaway radiation losses to occur, and the latter are abetted so that the cool condensation of a solar prominence is formed? How can the stressed magnetic field of an active region be reconnected quickly enough, in the highly conducting corona, to explain the short time scales of a solar flare? How are coronal mass ejections launched against the gravitational and magnetic forces of the lower atmosphere so that they can escape into the solar wind?

The UCI group has made significant progress on each of these problems, as will be described in the following sections. The key to our attack on the coupled dynamics and energetics of such nonuniform, anisotropic, nonlinear, active phenomena is the applica-

tion of large-scale numerical simulations, balanced and supported by analytic calculations and perturbation computations.

## Coronal Heating

One of the outstanding mysteries of solar physics is how the corona is able to sustain its high temperature while the chromosphere below, separated only by a narrow transition region, has a temperature two orders of magnitude lower. Energy must be transported from the solar surface to the corona by mechanisms other than thermal conduction, or large-scale convection, which is not present in the transition region.

Because the corona is threaded by magnetic fields of various intensities, Alfvén waves provide a promising medium of energy transport since they are naturally excited by photospheric fluid motions driven by subsurface convection (Ionson, 1978). However, the energy-deposition rate of classic Alfvén waves in a uniform magnetic field is known to be insufficient by many orders of magnitude; thus we have explored the characteristics of these waves in various nonuniform magnetic structures believed to be present in the solar atmosphere and have recalculated their energy-deposition rates. The effects of classical dissipation mechanisms such as resistivity (Mok and Emswiler, 1985; Emswiler and Mok, 1985, 1987) are enhanced by several orders of magnitude by the field nonuniformities existing in the corona. This is because fluid motions change sharply over a narrow resonant layer located in the magnetic inhomogeneity. The large magnetic-perturbation gradients, associated with the sharp velocity changes and maintained by the wave motions outside the layer, induce a strong current that dissipates the wave energy through resistivity. The dissipation of kinetic energy by viscosity is also enhanced in a similar manner. As a result, the energy deposition due to Alfvén waves is now believed to be significant in the process of coronal heating.

A second mechanism proposed for coronal heating (Parker, 1983) involves the generation of very narrow current layers ("sheets," in Parker's view) as a result of the convective/vortical motions of the surface footpoints of the coronal field lines. We have performed a three-dimensional MHD simulation of this process (Mikic et al., 1989), which evolves the coronal magnetic field from Parker's uniform initial state. Late-stage contours of the current density halfway along the field structure are shown in Figure 1. A diagnosis of this randomly driven evolution demonstrates that the magnetic field gradients grow exponentially, rather than discontinuously. This result

verifies an analytic prediction by van Ballegooyen (1988). As the current densities increase because of the convective driver, their ohmic losses ( $n^2$ ) also grow, until the limit at which the total Poynting-flux surface input is dissipated in the corona.

We have also studied the problem of the cross-field transport of the coronal heat input from the typical narrow energy-deposition filaments (as in the example just described) to the wider atmosphere. One promising process involves ion-cyclotron turbulence, which has the lowest drift-current threshold (and thus is also a contributor to the primary heating) and propagates substantially across the field. Auer and Benford (1989) have shown that ion-cyclotron waves can effectively spread the heat input over  $10^2$ – $10^3$  times the initial heat-input volume.

## Thermal and Filamentation Instabilities

Prominences are relatively dense, gaseous structures extending many thousands of kilometers into the Sun's atmosphere. They typically form in a few hours and can endure for many weeks. Although prominences were observed in ancient times, the physical process responsible for their formation has remained a mystery. It has been proposed that prominences result from a radiative-condensation instability (Field, 1965) that occurs when heat flow is inhibited by solar magnetic fields (Chidister and Van Hoven, 1979). When the fields are twisted (as indicated by increased fibril inclination), the dominant parallel heat flow ( $\kappa_{\parallel}/\kappa_{\perp} > 10^{10}$ ) is suppressed by the increased boundary distance along the field. The condensation instability is driven by temperature-dependent, optically thin radiation ( $\propto \rho^2 \phi(T)$ ) from highly ionized, heavy ion lines. When a dense cooling perturbation loses more energy through radiation ( $d\phi/dT < 0$ , initially) than it gains through thermal conduction or adiabatic and nonadiabatic heating processes, the local temperature and pressure drop. This results in additional mass inflow along the magnetic field, leading to increased density and runaway cooling.

The UCI group has been engaged in a long-term study of the linear and nonlinear condensational instability in the presence of magnetic shear. Linear studies (see Sparks and Van Hoven, 1985, 1988, and references cited therein) have distinguished between dynamic condensations, modes whose spatial structure is determined primarily by force balance, and kinematic condensations, where considerations of energy balance (especially perpendicular conduction over small scales) are paramount in setting characteristic scales of spatial variation. It is the latter, which do not exist when  $\kappa_{\perp} = 0$ , that are of greatest interest in explaining prominence formation. For such short-wavelength  $k_{\perp} \neq 0$  modes (Drake et al., 1988) plasma flow is primarily parallel to the magnetic field. Hence the field does not impede plasma compression, and significant density enhancement can occur. Kinematic modes also have nearly degenerate growth rates and quite small scales in both cross-field directions (Van Hoven et al., 1986). Thus they provide a natural explanation for the complex fine structure observed in prominences.

To study the nonlinear evolution of the

## Turbulent excitation of spontaneous reconnection

By D. DEEDS AND G. VAN HOVEN

Department of Physics, University of California, Irvine, California 92717 U.S.A.

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We explore the long-term nonlinear evolution of a tearing-mode-unstable sheared-field plasma in a turbulent environment. Two different physical configurations are modeled, and a different computational system is used for each. Results of both sets of calculations show that magnetic tearing arises spontaneously provided that the initial turbulence energy level is below the natural saturation level of the tearing instability.

We discuss briefly the relationship between our results and those of previous calculations, concluding that there are no significant unexplainable disagreements.

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### 1. Introduction

There has been considerable interest, in various communities, in turbulence and its relation to magnetic reconnection or tearing. Turbulence is perceived as a fundamental state of a wide variety of physical systems, with recent studies in 'chaos' providing some insight into the structure and development, as well as the description, of turbulent behaviour. In short, we can expect just about any realistic complex system to exist in a turbulent environment; thus we need to understand the consequences of this condition. From a more immediate point of view, many hydromagnetic phenomena in plasma physics are found in circumstances where turbulence is known to occur: for example, in tokamaks and other confinement devices, geomagnetic substorms, the solar corona (and presumably also the less observable regions of the sun). Quite reasonably, there have been efforts to understand puzzling effects, such as disruptions of confinement and anomalously fast solar-flare magnetic-energy release rates, in terms of turbulent reconnection and its consequences. This interest in turbulent effects on reconnection has been manifested in theory and in numerical simulation. Among the analytical work of interest in the present context has been that of Biskamp & Welter (1983), who claimed that under certain assumptions it could be shown that an anomalous resistivity and viscosity arose from excesses of magnetic or kinetic energy in the turbulent spectrum. Diamond *et al.* (1984) have also derived formal expressions for turbulently modified resistivity and viscosity, and Strauss (1986) has identified a hyper-resistivity, depending on the spectral composition of turbulence and having spatial extent, as well as an anomalous resistivity also dependent on the turbulence spectrum but effective only at the tearing layer. Working geometries are either slab or toroidal, the latter usually being approximated at some stage as locally cylindrical.



## GENERATION OF FINE-SCALE STRUCTURE IN THE SOLAR MAGNETIC FIELD

Zoran Mikić and Dalton D. Schnack

Science Applications International Corporation, 10260 Campus Point  
Drive, San Diego, CA 92121, U.S.A.

Gerard Van Hoven

Department of Physics, University of California, Irvine, CA 92717,  
U.S.A.

### 1. INTRODUCTION

The temperature of the solar atmosphere rises from 6000 K at the photosphere to 1-2 million degrees in the corona. This paradoxical temperature increase implies the existence of a coronal heating source. The energy loss mechanism (principally conduction) in the corona requires a heating source of between  $10^6$  and  $10^7$  erg/cm<sup>2</sup>-sec, depending on the region considered, to sustain the corona. Several explanations have been advanced to account for this heat source, including the dissipation of waves incident from below the photosphere, and the direct dissipation of magnetic field energy. Detailed studies of wave dissipation have shown that it is difficult to heat the upper corona by this mechanism. We consider a mechanism in which the magnetic field acts as an intermediary in converting mechanical energy present in photospheric convection into heat energy in the corona.

The convective motions in the photosphere can twist and braid the large-scale solar magnetic field. This induces electric currents to flow in the corona. It has been suggested that resistive dissipation of these currents can heat the corona<sup>1</sup>. The photospheric motions have a characteristic length scale  $l_p \sim 10^6$  m. Since the high-temperature corona is highly conducting, the magnetic field needs to acquire fine-scale spatial structure (on the length scale  $l_f \sim 10$ -100 m) to provide an effective heat source through resistive dissipation.

Parker<sup>2,3</sup> has described a scenario in which footpoint motions produce a loss of equilibrium and lead to the formation of current sheets. Here we investigate how photospheric flows may induce fine-scale structure in the magnetic field as a result of a cascade of

## THE EFFECT OF MASS LOADING ON THE TEMPERATURE OF A FLOWING PLASMA

 Jon A. Linker<sup>1,2</sup>, Margaret G. Kivelson<sup>1</sup> and Raymond J. Walker

Institute of Geophysics and Planetary Physics, University of California, Los Angeles

**Abstract.** We have investigated how the addition of ions at rest (mass loading) affects the temperature of a flowing plasma in a magnetohydrodynamic (MHD) approximation, using analytic theory and time dependent, three-dimensional MHD simulations of plasma flow past Io. The MHD equations show that the temperature can increase or decrease relative to the background, depending on the local sonic Mach number ( $M_S$ ) of the flow. For flows with  $M_S > \sqrt{9/5}$  (when  $\gamma = 5/3$ ), mass loading increases the plasma temperature. However, the simulations show a non-linear response to the addition of mass. If the mass loading rate is large enough, the temperature increase may be smaller than expected, or the temperature may actually decrease, because a large mass loading rate slows the flow and decreases the thermal energy of the newly created plasma.

## Introduction

The term "mass loading" refers to the addition of mass to a flowing plasma by ionization of neutrals. Io, Jupiter's innermost Galilean satellite, is embedded in a flowing plasma (the Io torus) and is also surrounded by a cloud of neutral atoms and molecules [Brown et al., 1983, and references therein]. In this paper we present results from three-dimensional MHD simulations of plasma flow past Io, including the effect of ionization of neutrals in the equations. Ionization may modify the plasma temperature significantly. We demonstrate these effects by examining the MHD equation for temperature and the results of simulation runs for three different values of  $M_S$ .

Mass loading plays an important role not only in Io's interaction with the plasma torus, but also in the solar wind interaction with comets and with Venus, in plasma flow past Saturn's satellite Titan, and possibly in other plasma-satellite interactions. Although the simulation parameters we use in this paper correspond most closely to the Io torus parameter regime, our remarks regarding the plasma temperature (in an MHD approximation) apply to the plasma in these other systems as well.

## The MHD Equations and the Plasma Temperature

We use the time-dependent MHD equations, including the effect of neutrals ionized near Io. When Jupiter's gravity is accounted for, the escape velocity for neutrals from the surface of Io is about 2.3 km/s [Linker et al., 1985] and the neutral density in Io's exosphere is dominated by lower velocity neutrals on non-escape trajectories [Watson, 1981; Sieveka and Johnson, 1984; Linker

1987; McGrath and Johnson 1987]. The torus plasma flows at 57 km/s relative to Io, so on average, the neutrals can be assumed to be at rest relative to Io. With that assumption, the normalized MHD equations can be written in the form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = \dot{\rho}_s \quad (1)$$

$$\frac{\partial}{\partial t} (\rho \vec{v}) + \nabla \cdot \left( \rho \vec{v} \vec{v} + \left( P + \frac{B^2}{2} \right) \mathbf{I} - \vec{B} \vec{B} \right) = 0 \quad (2)$$

$$\frac{d}{dt} (P \rho^{-\gamma}) = \frac{\gamma-1}{\rho^\gamma} \left( \eta J^2 + \frac{\dot{\rho}_s v^2}{2} - \frac{\gamma P \rho^{-\gamma} \dot{\rho}_s}{\rho} \right) \quad (3)$$

$$\frac{\partial \vec{B}}{\partial t} - \nabla \times (\vec{v} \times \vec{B}) = -\nabla \times (\eta \vec{J}) \quad (4)$$

where  $\rho$  is the density,  $\vec{v}$  the velocity,  $P$  the plasma pressure,  $\vec{B}$  the magnetic field,  $\eta$  the resistivity, and  $\vec{J} = \nabla \times \vec{B}$  the current density.  $\dot{\rho}_s$  is the mass added per unit volume per unit time. We take  $\gamma = 5/3$ . (Viscous terms are added to (2) and (3) in the simulation runs for numerical stability purposes, but they are irrelevant to the present discussion.) Equation (3) can be derived in a straightforward manner, using (1) and an equation for the conservation of energy (note that  $d/dt$  is a convective derivative) [Linker, 1987]. Because we consider that ionization of neutrals results from electron-impact, there should also be a heat loss term in (3) representing the energy given up by the electrons. However, the flow energy gained by an oxygen ion created in the background flow (280 eV) is large compared to the energy lost by the electron (10-12 eV). The inclusion of the electron energy loss term did not alter the simulation results significantly; therefore, we have ignored the term.

From (1-3) we can also derive equations for the plasma velocity and plasma pressure similar to those used by Schmidt and Wegmann [1982] and Ogino et al. [1988]:

$$\rho \frac{d\vec{v}}{dt} = -\nabla \cdot \left( \left( P + \frac{B^2}{2} \right) \mathbf{I} - \vec{B} \vec{B} \right) - \dot{\rho}_s \vec{v} \quad (5)$$

$$\frac{\partial P}{\partial t} + \nabla \cdot (P \vec{v}) = (\gamma - 1) \left( -P (\nabla \cdot \vec{v}) + \eta J^2 + \frac{1}{2} \dot{\rho}_s v^2 \right) \quad (6)$$

We see from (1), (5), and (6) that mass loading causes increases in the plasma density and pressure, and decreases in the plasma velocity. In the context of the Io torus, it has been noted that mass loading can increase the plasma temperature [e.g., Goertz, 1980; Brown et al., 1983; Shemansky, 1988]. However, if we assume an ideal gas law ( $P = \rho R T$ ), (6) implies that the plasma temperature,  $T$ , satisfies

$$\frac{dT}{dt} = (\gamma - 1) \left( \frac{\frac{1}{2} \dot{\rho}_s v^2}{\rho R} - T (\nabla \cdot \vec{v}) + \frac{\eta J^2}{\rho R} \right) - \frac{\dot{\rho}_s T}{\rho} \quad (7)$$

<sup>1</sup>Also at Department of Earth and Space Sciences, University of California, Los Angeles.

<sup>2</sup>Now at Physics Department, University of California, Irvine.

## THE NONLINEAR EVOLUTION OF MAGNETIZED SOLAR FILAMENTS

L. SPARKS<sup>1</sup> AND G. VAN HOVEN

Department of Physics, University of California, Irvine

AND

D. D. SCHNACK

Science Applications International Corporation

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### ABSTRACT

Thermal instability driven by optically thin radiation is believed to initiate the formation of plasma filaments in the solar corona. The fact that filaments are observed generally to separate regions of opposite, line-of-sight, magnetic polarity in the underlying, differentially rotating photosphere suggests that filament formation requires the presence of a highly sheared, local magnetic field. We have performed two-dimensional, nonlinear, magnetohydrodynamic simulations of the local genesis and growth of solar filaments in a force-free, sheared, magnetic field. To clarify the essential physics of the nonlinear behavior, we have traced the evolution of generic perturbations possessing broad spatial profiles. We find that many of the physical considerations that govern the behavior of linear condensations play a similar role nonlinearly. Simulations of the evolution of initial random-noise perturbations produce filamentary plasma structures that exhibit densities and temperatures characteristic of observed solar filaments. Furthermore, in each of these simulations, the filament axis lies at a finite angle with respect to the local magnetic field, consistent with solar observations.

*Subject headings:* hydromagnetics — Sun: magnetic fields — Sun: prominences

### 1. INTRODUCTION

Observations of the solar disk in H $\alpha$  often show thin, dark lines extending distances of 200,000 km or more. Such structures are called solar filaments. They consist of cool, dense plasmas suspended in the solar atmosphere above the photosphere. When viewed above the limb, they appear as prominences. Magnetograms and associated H $\alpha$  observations of fibrils reveal that solar filaments generally arise in regions where the local magnetic field is twisted and strongly varying. Observations have revealed that the axis of a fully formed prominence lies typically at a small angle (whose distribution is centered on 22.5°) with respect to the local magnetic field (Leroy 1989).

The processes responsible for the formation of solar filaments are not well understood. It is widely believed that filamentary structures form initially as a consequence of a local, thermal instability in the corona (Parker 1953; Field 1965). This mechanism, designated "the condensational instability," is driven by optically thin radiation. In thermal equilibrium, radiative energy losses are balanced by adiabatic and non-adiabatic heating and thermal conduction. Perturbing the plasma can upset this energy balance. When a dense plasma perturbation allows radiative cooling to dominate, the local pressure drops. This results in additional mass inflow and runaway cooling.

Thermal conduction tends to inhibit the condensational instability (Orrall and Zirker 1961). Indeed, it is well known that heat flow (mediated largely by electron collisions) in an unmagnetized coronal plasma is so rapid that the condensational instability is completely suppressed for all temperature (and density) perturbations whose transverse scales are less

than a limit somewhat larger than the size of a typical filament. Consequently, the ability of a magnetic field to reduce (electron) heat flow by many orders of magnitude (in directions perpendicular to the field) is essential for the formation of observed coronal condensations.

The presence of a magnetic field, however, affects the plasma dynamics as well as the energetics. Due to the high electrical conductivity and the low  $\beta$ -value of the ambient corona, plasma fluid elements are tied to magnetic field lines, and mass flow tends to be directed parallel to the magnetic field—the direction in which heat flow is not affected by the field. *How is it possible for a magnetic field to inhibit heat flow in a coronal plasma without simultaneously restricting the mass flow required for the growth of density condensations?* This is, perhaps, the central question facing attempts to attribute the initiation of filament formation to the condensational instability, and this is the principal question we wish to address in this paper.

It has been suggested that filament formation may require the presence of a magnetic field  $B(x)$  that is locally *sheared* (Chiuderi and Van Hoven 1979). Coronal condensations with densities and spatial scales characteristic of solar filaments cannot form in a *uniform* magnetic field. Such a field prevents transverse mass flow from contributing to a local growth in density, while heat flow parallel to the field ensures that parallel mass flow will not give rise to condensational instability. Observational evidence supports the conclusion that shear in the magnetic field is a necessary local condition for filament formation. Magnetograms reveal that filaments typically separate regions of the Sun possessing opposite, line-of-sight, magnetic polarity, and H $\alpha$  observations demonstrate that filament formation is often initiated after a period of divergent fibril inclination (Martin 1973).

The anisotropic influence of local, force-free magnetic shear on the *linear* dynamics of the condensation instability has been examined extensively in a series of studies (see Sparks and Van

<sup>1</sup> Present address: Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California.

# MHD Simulations of Coronal Mass Ejections: Importance of the Driving Mechanism

J. A. LINKER AND G. VAN HOVEN

*Department of Physics, University of California, Irvine*

D. D. SCHNACK

*Science Applications International Corporation, San Diego, California*

We have investigated the importance of the form of the driving mechanism in MHD simulations of coronal mass ejections. Previous authors have performed simulations using a thermal driving mechanism, and have found that this mechanism cannot reproduce the observed features of mass ejections unless an elaborate model of the initial corona is used. We have devised a model simulation problem and have found that the use of a simple form for the initial corona, with an upward moving parcel of cold, dense plasma as the driving mechanism, can produce results that are consistent with many of the features observed by coronagraphs. Our results imply that the nature of the driving mechanism may play an important role in determining the dynamical evolution of mass ejections.

## INTRODUCTION

Coronal mass ejections (CMEs) are important but poorly understood phenomena in solar terrestrial physics. CMEs have been observed in space-based white-light coronagraphs since the early 1970s, yet fundamental questions about their initiation and propagation in the solar corona remain unanswered.

About 1/3 of the CMEs seen in the Skylab data, and as many as 80% of those observed in the Solar Maximum Mission (SMM) data below 3  $R_{\odot}$  (solar radii) can be classified as "looplike" [Wagner, 1984]. Sime et al. [1984] identified three general characteristics of the looplike mass ejections observed with the Skylab coronagraph: (1) the sides of the loop (often referred to as the "legs" of the mass ejection) have a greater density enhancement than the top of the loop; (2) a depletion of density occurs between the legs of the mass ejection; (3) the legs of the mass ejection exhibit very little lateral motion throughout the time evolution of the mass ejection. In this paper our references to CME observations are specifically to the looplike ejections of the form that Sime et al. describe.

Coronal mass ejections are complex phenomena, and a realistic model must involve at least two spatial dimensions, time dependence, and magnetic forces. One approach for studying such phenomena are time-dependent magnetohydrodynamic (MHD) simulations. The earliest attempts at numerical simulation of CMEs explored the possibility that mass ejections could be initiated by the rapid injection of thermal energy at the base of the corona from a solar flare [e.g., Steinolfson et al., 1978; Dryer et al., 1979; Wu et al., 1982]. In these simulations, the initial corona was modeled with a potential (current-free) magnetic field, and a pressure and density satisfying hydrostatic equilibrium. A pressure pulse was

introduced to model the thermal energy released in the flare, and the subsequent time evolution was followed. The resulting expanding fast-mode shock wave was identified as the CME. However, Sime et al. [1984] pointed out that these simulations failed to replicate the three (previously listed) observed characteristics of looplike ejections. Because of these discrepancies, Sime et al. questioned the idea that CMEs could be regarded as compressional disturbances initiated by thermal energy release. We note that Dryer and Wu [1985] disputed this conclusion, as well as the generality of the CME characteristics identified by Sime et al.

Independent of the observations of Sime et al., another difficulty with the idea that CMEs are driven by thermal energy release from solar flares arises from the observation that mass ejections are more often associated with eruptive prominences than with flares [Kahler, 1987, and references therein]. The eruption of a prominence and the deduced departure time of an associated CME have been found to be roughly simultaneous, whereas in flare-associated ejections, extrapolation of the observations suggests that the flare is initiated by conditions occurring after the launch of the mass ejection [Wagner et al., 1983; Harrison, 1986].

As one response to the problems with the thermal energy driving mechanism, Steinolfson [1988] and Steinolfson and Hundhausen [1988] have taken an approach that emphasizes the role of the initial corona. They first verified that simulations with a thermal driving mechanism and an initially hydrostatic, current-free corona with a closed magnetic topology could not reproduce any of the observed features of mass ejections. They then went on to show that a thermal driving mechanism can simulate many of the observed features of coronal mass ejections, if one is able to specially tailor the initial corona. A streamer-like configuration was used, and a heating term was added to the energy equation to increase the background sound speed in the calculation.

The work by Steinolfson and Hundhausen [1988] was a useful step in understanding CME evolution, in that they showed one way of qualitatively reproducing the

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# Magnetohydrodynamic modeling of the solar corona\*

Zoran Mikić

Science Applications International Corporation, 10260 Campus Point Drive, San Diego, California 92121

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The ideal and resistive magnetohydrodynamic (MHD) model is used to examine the dynamics and structure of the solar corona. When the coronal magnetic field is deformed by photospheric flow it can evolve to states that become unstable to ideal MHD modes. The nonlinear evolution of these instabilities can lead to the generation of current sheets, field line reconnection, and energy release. The disruption of an arcade field and the kinking of coronal loops is described. The braiding of the large-scale coronal field by convective photospheric motions develops fine-scale structure in the magnetic field and leads to the development of intense current filaments. The resistive dissipation of these currents can provide an efficient coronal heating mechanism.

## 1. INTRODUCTION

The solar corona presents an environment that challenges the ingenuity of the plasma physicist. The combination of the slow evolution of magnetic fields in three-dimensional geometry and the occurrence of impulsive energetic events that are observed during solar flares makes the analysis of coronal plasmas difficult. The hot, highly conducting corona has widely disparate Alfvén and resistive time scales: the Lundquist number  $S = \tau_R / \tau_A$ , where  $\tau_A$  is the Alfvén time and  $\tau_R$  is the resistive diffusion time, is of order  $10^{12}$ . The simultaneous large separation of length scales (global field structures  $\sim 10^4$  m, current filaments  $\sim 10^2$  m) complicates the analysis further. A description of the corona based on heuristic, severely approximate qualitative models gives necessarily vague conclusions about its properties. Recent developments in algorithms for the numerical solution of the magnetohydrodynamic (MHD) equations have enabled us to follow the dynamical evolution of idealized coronal magnetic fields in realistic geometry. Such MHD simulations have become indispensable in developing an understanding of coronal physics. We describe the application of three-dimensional, time-dependent, ideal and resistive MHD simulations to global coronal behavior. In particular, we describe the disruption of an arcade field that is sheared by photospheric flow, we investigate the nonlinear development of the kink instability in coronal loops, and we show how the braiding of the coronal magnetic field by smooth, random photospheric flows can introduce fine-scale structure into the coronal field. The dissipation of the electric current filaments so introduced can act as an effective coronal heating mechanism. The application of the MHD model to coronal magnetic fields is discussed by Parker,<sup>1</sup> Priest,<sup>2</sup> and Birn and Schindler.<sup>3</sup> Also see a recent article by Low.<sup>4</sup>

The crucial difference between coronal and laboratory plasmas is that field lines in the corona are rooted in the photosphere (line tied). Photospheric motions cause coronal field structures to evolve quasistatically through sequences of neighboring equilibria. Certain motions can drive

the system across a stability threshold, leading to impulsive dynamical behavior. The practical simulation of this slow evolution, coupled with impulsive nonlinear behavior, requires the use of efficient numerical methods. The semi-implicit numerical method<sup>5,6</sup> is suitable for this problem. This technique allows a large time step to be used during the quasistatic evolution phase, which involves long-wavelength phenomena. The time step is reduced automatically during the more dynamic phases of the nonlinear evolution (current filament formation, reconnection, and front propagation) for accuracy.

The MHD model provides an accurate description of global coronal physics because coronal phenomena are characterized by large length scales and long time scales (compared to the ion gyroradius and gyroperiod, respectively). Kinetic models are still necessary, though, to describe the phenomena of x-ray and particle radiation that accompany flares. Although the plasma resistivity is generally negligibly small, it becomes crucially important in regions of concentrated electric current density. Thus we use the resistive MHD model,

$$\nabla \times \mathbf{B} = (4\pi/c)\mathbf{J}, \quad (1)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (2)$$

$$\mathbf{E} + (1/c)\mathbf{v} \times \mathbf{B} = \eta \mathbf{J}, \quad (3)$$

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \frac{1}{c} \mathbf{J} \times \mathbf{B} - \nabla p + \nu \rho \nabla^2 \mathbf{v}, \quad (4)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (5)$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p = -\gamma p \nabla \cdot \mathbf{v}, \quad (6)$$

where  $\mathbf{B}$  is the magnetic field,  $\mathbf{J}$  is the electric current density,  $\mathbf{v}$  is the plasma velocity,  $\mathbf{E}$  is the electric field,  $p$  is the plasma pressure,  $\rho$  is the mass density,  $\eta$  is the resistivity,  $\nu$  is the viscosity, and  $\gamma = \frac{5}{3}$ . The ideal MHD model is obtained by setting  $\eta = 0$ . Line tying at the photosphere is implemented by using an ideal Ohm's law at the boundaries corresponding to the photosphere,

$$\mathbf{E} + (1/c)\mathbf{v}_p \times \mathbf{B} = 0, \quad (7)$$

\*Paper 812, Bull. Am. Phys. Soc. 34, 2138 (1989).

# PROMINENCE FORMATION IN A CORONAL LOOP

Y. MOK, J. F. DRAKE, D. D. SCHNACK, AND G. VAN HOVEN

Department of Physics, University of California, Irvine

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## ABSTRACT

We present a model, which depends on the preferential deposition of heating in the legs of a coronal loop, that produces a stable prominence-scale condensation at the loop top. Dynamic stability is attained by the subsequent adjustment of local parallel gravity by a magnetic inversion at the loop (or arcade) apex. We describe a nonlinear numerical simulation of this process, which includes a deep chromosphere, a heating rate with a fixed dissipation length, and full solar gravity.

**Subject headings:** hydrodynamics — Sun: atmosphere — Sun: corona — Sun: prominences

## 1. INTRODUCTION

One of the unsolved problems of solar physics is the character of the energy-balance and mass-transfer mechanisms by which a prominence can cool and condense at the apex of a magnetic arcade or loop. The elements involved include sufficient local densification or suppression of parallel heat conduction to allow radiative cooling to dominate the energy transport (Field 1965), and sufficient pressure gradients to pull material along the field from the chromospheric source (Hildner 1971), against solar gravity, to the coronal apex. One attempt to provide these conditions in a multistep process using a fixed gravitational well was described by Poland and Mariska (1986). A related, continuous, nonlinear process was recently detailed by Sparks, Van Hoven, and Schnack (1990), in the absence of gravity and with the chromosphere modeled by inflow boundaries.

In this paper we describe a single-step or continuous mechanism for providing a cool condensation which is then confined in a gravitational well located at the apex of a coronal loop or arcade.

## II. MODEL AND EQUATIONS

We begin by assuming that the system is confined by a strong magnetic field, so that the mass flow and heat flux are essentially one-dimensional. This assumption is valid in most coronal loops and magnetic arcades observed in the corona, which have field lines anchored in the photosphere. The evolution of this system is described by the equations of hydrodynamics, supplemented by an energy equation to take into account the effects of radiative loss, plasma heating and parallel thermal conduction. The energy equation can be written as

$$2nk\left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x}\right) = -2nk(\gamma - 1)T \frac{\partial v}{\partial x} + (\gamma - 1)\left[H - n^2\phi(T) + \frac{\partial}{\partial x} \kappa \frac{\partial T}{\partial x}\right] \quad (1)$$

where  $v$  is the fluid velocity,  $T$  the temperature,  $n$  the electron number density,  $H$  the heating function,  $n^2\phi(T)$  the average radiative loss rate per unit volume,  $\gamma$  the ratio of specific heats,  $k$  Boltzmann's constant, and  $\kappa(T)$  the (parallel) thermal conductivity. In our model,  $\kappa \sim T^{5/2}$  is taken to be the classical value, although other nonclassical models can easily be implemented in the present calculation.

The radiation law  $\phi(T)$  is that given by Athay (1986), modified by an imposed cut-off below  $2 \times 10^4$  K due to Ly $\alpha$  and other atomic lines which make the plasma optically thick at lower temperatures. The heating function is based on a model in which energy is supplied to the chromospheric and coronal plasma from the photosphere through various channels, such as Alfvén waves, acoustic waves, etc. Since the detailed heating mechanism and its characteristics are not yet fully understood, we will treat the present heating function in a phenomenological way as a working model. Accepting the fact that photospheric fluid motions are the source of energy, we argue that this supply naturally decays with altitude, as energy is deposited in the plasma along the upward path (see, for example, Priest and Smith 1979). The rate of decay, however, depends on the poorly understood heating mechanism and is treated as a variable parameter in the present calculation. In the actual computation, the heating function has the form  $H_0 \exp(-x/x_{\text{hd}})$ , where  $H_0$  is a constant,  $x$  is the distance from the base, and  $x_{\text{hd}}$  is the spatial decay-length of the heating source.

The magnetically collimated (1 - D) momentum equation is given by

$$\rho \left[ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right] = -\frac{\partial p}{\partial x} + \rho g \quad (2)$$

where  $\rho \approx nm_p$  is the mass density,  $p (= 2nkT)$  is the pressure, and  $g(x)$  is the component of the gravitational force along the magnetic field. In the present calculation,  $g(x)$  contains two contributions. The first one has the form  $-g_0 \cos(\pi z/l)$ , assuming that the confining magnetic field has a semicircular shape, where  $g_0 = 2.74 \times 10^4 \text{ cm s}^{-2}$  is the solar-surface gravity, and  $l$  is the total length of the loop. The second contribution is introduced to simulate the fact that the magnetic field will be distorted at the location where the condensation occurs (Kippenhahn and Schlüter 1957; Poland and Mariska 1986).

This distortion of the projected gravitational force is modeled (as the condensation forms) by the quantity  $g_1 \sin 2\pi[x - l_p]/[l - 2l_p]$ , with  $g_1$  the amplitude of the distortion in the range  $l_p < x < l - l_p$ , centered near the top of the loop structure, where condensation is expected because the heating reaches a minimum. In particular, this term is not turned on until a substantial condensation (with density  $\geq 5$  times the nearby coronal value) has formed. This contrasts with the approach used by Poland and Mariska (1986), whose

## DYNAMICAL EVOLUTION OF TWISTED MAGNETIC FLUX TUBES. I. EQUILIBRIUM AND LINEAR STABILITY

Z. MIKIĆ AND D. D. SCHNACK

Science Applications International Corporation, San Diego

AND

G. VAN HOVEN

Department of Physics, University of California, Irvine

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### ABSTRACT

The three-dimensional dynamical evolution of twisted magnetic flux tubes is studied using a time-dependent magnetohydrodynamic (MHD) model. The flux tubes are intended to model solar coronal loops, and include the stabilizing effect of photospheric line tying. The model permits the complete evolution of flux tubes to be followed self-consistently, including the formation, equilibrium, linear instability, and nonlinear behavior. Starting from an initial uniform background magnetic field, a twisted flux tube is created by the application of slow, localized photospheric vortex flows. The flux tube evolves quasi-statically through sequences of equilibria with increasing twist, until it becomes linearly unstable to an ideal MHD kink mode. In this paper the equilibrium properties and the linear stability behavior are discussed. The nonlinear behavior will be treated in the second paper of this series. The application of the method to the uniform-twist, Gold-Hoyle field confirms the previous stability threshold for kink instability and provides estimates of the resulting growth rate. When the method is applied to a loop with a localized twist profile, it is found that the loop becomes kink unstable when the twist exceeds  $4.8\pi$  on axis.

*Subject headings:* hydromagnetics — Sun: corona

### 1. INTRODUCTION

Twisted magnetic flux tubes can be detected in numerous observations of space plasmas. They are observed in the solar corona, and are likely to be present in the majority of plasma environments, including stellar coronae, planetary atmospheres, and the magnetosphere. It is believed that magnetic flux ropes influence the form and evolution of the Earth's magnetotail. Loop-like magnetic structures are ubiquitous in EUV and X-ray images of active regions of the solar corona. They play a crucial role in the structure, dynamics, and heating of the solar corona. In this paper we emphasize the behavior of solar coronal loops, although many of the properties we investigate are generally applicable to twisted magnetic flux tubes. For a review of recent theoretical and observational results on the behavior of magnetic flux ropes, see Russell, Priest, and Lee (1990).

Coronal loops consist of magnetic flux tubes which can result either from the emergence of bundles of twisted field structures from beneath the photosphere, or can be formed from the ambient coronal magnetic field by the twisting motions inherent in subphotospheric convection. The large photospheric density and the small value of the plasma resistivity create a situation of almost "frozen flow," in which the footpoints of the coronal magnetic field lines which penetrate the photosphere are convected by the photospheric flow. It has long been recognized (Gold and Hoyle 1960) that coronal loops are likely to be unstable to kink modes if they become too twisted. It is well known from laboratory-plasma theory and experiments that pinch and tokamak discharges become unstable to ideal magnetohydrodynamic (MHD) kink modes if the toroidal plasma current (which is proportional to the twist of the field) exceeds a threshold amount (Kruskal *et al.* 1958; Shafranov 1957).

It was, however, recognized that on the Sun photospheric line tying provides a stabilizing influence which allows coronal loops to be stable up to higher levels of twist than corresponding laboratory plasmas (Raadu 1972; Giachetti, Van Hoven, and Chiuderi 1977; Einaudi and Van Hoven 1981). Hood and Priest (1979, 1981) and Einaudi and Van Hoven (1983) have computed stability thresholds for the ideal MHD kink mode in coronal loops which explicitly demonstrate the stabilizing effect of photospheric line tying. Recently, Velli, Einaudi, and Hood (1990a) have analyzed the linear, ideal MHD stability of kink modes in coronal loops, and have presented a technique for the investigation of resistive instabilities (Velli, Einaudi, and Hood 1990b).

It is of interest to determine the dynamical consequences of kink instability in coronal loops, especially as they affect observable coronal phenomena. We describe a method which is employed in the investigation of the temporal evolution of twisted flux tubes. We advance the three-dimensional MHD equations numerically in time, using ideal and resistive models, to simulate the dynamics of the coronal plasma. Our model allows us to self-consistently follow the complete history of the evolution of flux tubes, including the formation, equilibrium, linear instability, and nonlinear behavior. Starting from an initial uniform background magnetic field, a twisted flux tube is created by the application of slow, localized photospheric vortex flows. For twists beyond a critical threshold, flux tubes become linearly unstable to ideal or resistive kink modes. The nonlinear evolution of these kink instabilities may generate concentrated current filaments. The resistive dissipation of these current filaments may provide a heating source for the corona. In this paper we discuss the equilibrium properties and the linear stability behavior. The nonlinear behavior will be treated in the second paper of this series.

# FILAMENT COOLING AND CONDENSATION IN A SHEARED MAGNETIC FIELD

Gerard Van Hoven

Department of Physics, University of California, Irvine, California 92717

**Abstract.** Thermal instability driven by optically thin radiation in the corona is believed to initiate the formation of solar filaments [Parker, 1953]. The fact that filaments are observed generally to separate regions of opposite, line-of-sight, magnetic polarity in the differentially rotating photosphere suggests that filament formation requires the presence of a highly sheared magnetic field. In this paper we discuss the coupled energetics and dynamics of the most important condensation modes, those due to perpendicular thermal conduction at short wavelengths. We describe their linear structure in the sheared field and their growth rates. We have also performed two-dimensional, nonlinear, magnetohydrodynamic simulations of the evolution of these modes in a force-free field. To clarify the essential physics of the nonlinear behavior, we have traced the evolution of generic perturbations possessing broad spatial profiles. The simulations achieve the fine thermal structures, minimum temperatures and maximum densities characteristic of observed solar filaments.

The anisotropic influence of magnetic shear on the linear dynamics of the condensation instability has been examined extensively in a series of studies [Chiuderi and Van Hoven, 1979; Van Hoven and Mok, 1984; Van Hoven et al., 1986; and Sparks and Van Hoven, 1988]. Several conclusions have emerged from these studies:

1. A condensation will form preferentially in regions in which  $k_{\parallel} \equiv \mathbf{k} \cdot \mathbf{B}/B \approx 0$  where  $\mathbf{k}$  represents the (local) wave vector of the perturbation.
2. A condensation may be classified according to whether its spatial structure is determined primarily by considerations of force balance or energy balance. The former is identified as a dynamic condensation, and the latter a kinematic condensation.

3. The modes that exhibit the fastest growth exist only in the presence of anisotropic heat flow.

4. Plasma mass flow within kinematic condensations is directed parallel to the magnetic field.

5. Kinematic condensations are most compressible when sound waves traveling parallel to the magnetic field can maintain pressure balance.

To study the evolution of solar filaments, we adopt a one-fluid, transport model [Van Hoven et al., 1987]. The dynamical behavior of the plasma is governed by the nonlinear equations of resistive magnetohydrodynamics, specifically including the continuity equation, the momentum equation, Maxwell's equations, and Ohm's law. This set of equations is closed by including an energy equation of the form

$$\frac{d\rho}{dt} = \gamma \frac{p d\rho}{\rho dt} + (\gamma - 1)[\nabla \cdot \kappa \cdot \nabla T - C + H] \quad (1)$$

where  $\kappa$  is the thermal conductivity tensor,  $C = R\rho^2 T^4$  denotes cooling due to radiative loss [Hildner, 1974], and  $H$  is a heating function. We specify the magnetic field by the model form  $\mathbf{B}_0 = B_0 [\text{sech}(y/a) \hat{\mathbf{e}}_x + \tanh(y/a) \hat{\mathbf{e}}_y]$  where  $a$  is the shear scale, which is in dynamic equilibrium with a uniform temperature and density, with an accompanying energetic balance of uniform heating and radiation. The MHD system can be reduced to a simultaneous set of seven equations relating the mass density  $\rho$ , the temperature  $T$ , the three fluid-velocity components  $u_x$ ,  $u_y$ , and  $u_z$ , the magnetic field component  $B_z$ , and the magnetic flux function  $\Psi(y, z)$ .

To understand the character of the initial plasma excitations that lead to significant cooling and condensation, we will consider a simplified model of the small-amplitude behavior of this system. To do so, we first linearize the MHD equations around the force-free, isobaric equilibrium just described by using  $T = T_0 + T_1(y) \exp(\nu t + i k z)$



## EFFECTS OF THE DRIVING MECHANISM IN MHD SIMULATIONS OF CORONAL MASS EJECTIONS

J. A. Linker and G. Van Hoven

Department of Physics, University of California, Irvine

D. D. Schnack

Science Applications International Corporation

**Abstract:** We present results of time-dependent MHD simulations of mass ejections in the solar corona. Previous authors have shown that results from simulations using a thermal driving mechanism are consistent with the observations only if an elaborate model of the initial corona is used. Our first simulation effort, using a simple model of a plasmoid as the driving mechanism and a simple model of the initial corona, produces results that are also consistent with many observational features, suggesting that the nature of the driving mechanism plays an important role in determining the subsequent evolution of mass ejections. Our first simulations are based on the assumption that mass ejections are driven by magnetic forces; we are now developing simulations where the initial corona is perturbed magnetically by introducing a "plasmoid-like" current perturbation. The preliminary results from these simulations show some features that are consistent with the observations, others that are not. The discrepancies may be caused by the lack of internal force balance in the initial plasmoid structure. In the future, we plan to perform simulations where plasmoid formation occurs self-consistently.

### Introduction

Coronal mass ejections (CMEs) have been observed in space-based white light coronagraphs since the early 1970s. Because of the complex nature of CMEs, their initiation and propagation in the solar corona is still not well understood. An important class of CME events are the looplike CMEs [Wagner, 1984]. Sime et al. [1984] identified three general characteristics of the looplike CMEs observed with the Skylab coronagraph: (1) the sides of the loop (the "legs" of the mass ejection) have a greater

density enhancement than the top of the loop, (2) a depletion of density occurs within the loop; (3) the legs exhibit very little lateral motion throughout the time evolution of the mass ejection.

One approach for investigating CMEs is the time-dependent magnetohydrodynamic (MHD) simulation. The first MHD simulations of CMEs were based on the assumption that mass ejections are initiated by the rapid injection of thermal energy at the base of the corona from a solar flare [e.g., Dryer et al., 1979; Wu et al., 1982]. In these simulations, the initial corona was modeled hydrostatically with a current-free magnetic field. A pressure pulse was introduced to model the thermal energy released in the flare, and the subsequent time evolution was followed. The resulting expanding fast-mode shock wave was identified as the CME. Although these simulations did give a coherent explanation for the initiation and propagation of CMEs, Sime et al. [1984] pointed out that these simulations failed to replicate the observed characteristics of looplike ejections. Because of these discrepancies, Sime et al. questioned the idea that CMEs could be regarded as compressional disturbances initiated by thermal energy release.

In order to obtain simulation results that are compatible with the observations, Steinolfson and Hundhausen [1988] examined the role of the initial corona. They first verified that simulations with a thermal driving mechanism and an initially hydrostatic, current-free corona with a closed magnetic topology could not reproduce any of the observed features of mass ejections. They then showed that if one used a specially tailored initial condition and a thermal driving mechanism, many of the observed features of CMEs could be reproduced. Their model assumes that the evolution of CMEs in the solar corona is primarily influenced by the pre-event corona, and that the nature of the driving mechanism does not greatly affect the dynamics.

## A SLINGSHOT MODEL FOR SOLAR FLARES

GREGORY BENFORD

Department of Physics, University of California, Irvine, CA 92717

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### ABSTRACT

Recent observations of intense, impulsive  $\gamma$ -ray- and X-ray-emitting solar flares underline the suddenness of these events. The simultaneous emission of X-rays  $>40$  keV from electron bremsstrahlung and  $\gamma$ -rays requiring several MeV protons shows that all particles must be accelerated in less than 5 s. This paper proposes a simple model to explain such events, using the energy stored in the stretched field lines of a coronal arch. When reconnection occurs at the top of the arch, field lines retract like stretched rubber bands, sweeping up plasma and acting like a piston or slingshot. When the slug of plasma caught in the magnetic fields strikes the photosphere, it deposits its considerable kinetic energy, heating and compressing the intruding slug. Ten slugs of 100 km radius striking the photosphere may account for the  $10^{29}$  ergs radiation from loop flares.

*Subject headings:* gamma rays: bursts — hydromagnetics — particle acceleration — Sun: flares — Sun: X-rays

### 1. INTRODUCTION

There is ample evidence that intense  $\gamma$ -ray- and X-ray-emitting solar flares require sudden processes, capable of yielding X-rays of 40 keV or more and  $\gamma$ -rays demanding protons of 10 MeV energies. Recent observations underline these constraints (Forrest & Chupp 1983; Nakajima et al. 1983; Ramaty et al. 1980). Earlier work assumed that electrons were accelerated first in solar flares (Švestka 1977; Brown & Smith 1980; Ramaty et al. 1980), yielding hard X-rays  $\sim 100$  keV. Slow acceleration processes then gave second-phase energetic ions, and heat conduction down from the reconnection site heated the feet of the arch.

Here we briefly propose that direct kinetic transport may be an important contributor to the overall energetics. If the relaxing magnetic field lines in a loop can directly accelerate plasma (Spicer 1977; Van Hoven 1979), this will lead to energy transfer which is quicker and more efficient than several-phase models. It can yield simultaneous emission throughout the X-ray and  $\gamma$ -ray spectrum from the same location at the arch feet.

The basic physical picture is of a cold, beamlike slug, ejected from the reconnection region. The slug thermal speed is less than its directed velocity, which is of the order of the Alfvén speed. This plasma strikes the photosphere, exciting ion-ion two-stream instabilities. These modes heat the slug to energies approaching a MeV, much faster than Coulomb collisions.

This paper sketches the basic energetics argument. Colliding plasmas may well suffer turbulent stopping processes, and these we discuss briefly, using a marginal instability estimate. Future analysis will take up the detailed problems of deceleration and thick-target emission.

### 2. MODEL

Figure 1 shows a coronal loop which has suffered some reconnection between field lines near the top. This may occur through convective twisting, or when buoyant field lines of opposite sign rise into the underside of an existing loop, reversing toroidal fields locally, so that reconnection occurs. A new arch can also emerge inside a preexisting arch of opposite polarity, although here we discuss primarily a single loop. Some heat is liberated near the reconnection, and the notable effect far from the site is an outflow of both plasma and mag-

netic fields at velocities near the Alfvén speed,  $v_A$  (Petschek 1964; Mok & Van Hoven 1982; Forbes & Priest 1987). This work showed that reconnection cannot occur without reversal of local toroidal field lines. We reflect this in the reversed configuration of Figure 1, which we assume can apply to a variety of distance scales. In loops, reversals at the footpoints are of size  $\sim 100$  km and not readily observable (Švestka 1981).

While the immediate reconnection energy release is considerable, much more available energy is stored in the field lines extended from the photosphere below. These can now contract, shortening the local field-plasma tube (Heyvaerts 1981).

This retracting tube will act like a slingshot, since magnetic forces dominate in the arch; the equilibrium ratio of plasma pressure to magnetic pressure,  $\beta_*$ , is  $3.5 \times 10^{-5} n_8 T_6 / B_2^2$ , where the plasma density  $n_8$  is in units of  $10^8 \text{ cm}^{-3}$ ,  $T_6 = (T / 10^6 \text{ K})$ , and  $B_2 = (B / 10^{-2} \text{ G})$ . These are typical values taken from observations (Švestka 1981). The vast magnetic field energy can convert to particle kinetic energy.

The force acting on a plasma slug is  $j \times B_*$ , where  $j$  is the current perpendicular to the plane of Figure 1 and  $B_*$  is the magnetic field perpendicular to the main toroidal field of the figure. We shall assume this component is considerably less than the equilibrium field, which is of order 100 G.  $B_*$  forms the cup of the slingshot.

What is the average ratio of thermal energy to magnetic energy in the slug,  $\beta$ , as it leaves the reconnection region? We assume an essentially force-free loop configuration, so there will be a field  $B_t$  perpendicular to the plane of Figure 1 and probably comparable in strength to the outside toroidal field. Reconnection will not annihilate this component, and it will resist compression of the slug plasma, setting a limit on heating from this compression. We shall take  $\beta < 1$ , but not necessarily very small, and certainly  $\beta > \beta_*$ , the equilibrium loop value.

The compressibility of the slug plasma is governed by

$$\beta_* \equiv \frac{8\pi nT + B_t^2}{B^2}$$

rather than simply  $\beta$ . We expect  $\beta_* \approx 0.1$ –1, because  $B_t < B$  for the equilibrium, but  $B_t$  can be of order  $B$ . This implies that contraction of field lines can increase kinetic energies of



## MAGNETOHYDRODYNAMIC SIMULATION OF CORONAL MAGNETIC FIELDS

D.D. SCHNACK, Z. MIKIĆ, D.C. BARNES

*Science Applications International Corporation, 10260 Campus Point Drive, San Diego, CA 92121, USA*

and

G. VAN HOVEN

*Department of Physics, University of California, Irvine, CA 92717, USA*

The application of supercomputers and advanced numerical techniques to problems of coronal structure and dynamics is described. Numerical methods appropriate for the long time scale simulation of nonlinear magnetohydrodynamic systems are discussed. Three specific examples of the application of these techniques to the solar corona are given. These are magnetic energy storage and conversion, a model for steady coronal heating, and calculation of stable force-free equilibria from given boundary data, such as that obtained with a vector magnetograph. It is suggested that the continued application of these methods will result in substantial advances in the understanding of coronal dynamics and structure.

### 1. Introduction

The solar corona, or outer atmosphere of the sun, consists of a hot, tenuous plasma permeated with a magnetic field. This magnetized plasma displays a rich variety of dynamical activity. The coronal magnetic field is rigidly attached to the photosphere, the visible solar surface. The footpoints of these field lines are constantly being moved about slowly by the motion of the photosphere, which is driven from below by convective processes. This motion produces a continual stirring of the coronal plasma. New magnetic flux emerges from below the photosphere and displaces plasma above it. Sunspots and their associated strong magnetic fields interact with each other, causing the magnetic field lines to slowly twist into loops and arcades resembling croquet hoops. Immense filaments of dense, cool plasma mysteriously form within the corona, and persist for periods of days or weeks. Occasionally these slow, churning processes yield to rapid, dramatic events. The local magnetic structure of the corona may be suddenly altered, with the release of vast amounts of energy in a very short period of time.

A broad band of radiative energy is released in the form of a solar flare. A cool, dense filament may erupt from the low corona. Rapid ejections of mass into interstellar space may accompany topological changes in the coronal magnetic field. These events may occur in concert or individually, and often seem to emerge spontaneously and without warning from the relatively quiescent corona.

A quantitative theoretical description of the dynamical corona is one of the main challenges of modern solar physics. It is generally believed that the source of the energy released in disruptive events such as solar flares, erupting prominences, and coronal mass ejections is the coronal magnetic field. The energy in this field has two components. One is due to electric current flowing below the photosphere. This energy can generally not be tapped as this would require the alteration of distant current sources. The other arises from electric currents flowing locally in the corona itself. This energy *can* be tapped by the local rearrangement of magnetic field. The origin of this free energy is likely the work done on the field by the slow motion of the photospheric surface. The central question is how large amounts of magnetic

## A Three-Dimensional Simulation of a Coronal Streamer

Jon A. Linker and Gerard Van Hoven

Physics Department, University of California, Irvine

Dalton D. Schnack

Science Applications International Corporation

**Abstract.** We have used a time-dependent magnetohydrodynamic (MHD) simulation to investigate the magnetic topology of an idealized streamerlike configuration in three dimensions. Starting from an initially current-free multipole field and a transonic flow, a quasi-steady, streamerlike configuration forms. The simulated streamer exhibits a closed magnetic field region bounded by a current layer and surrounded by open fields. This helmet region is surmounted by a current layer, with an enhanced density in the closed field and current sheet regions. The simulated streamer has a finite longitudinal extent, with the closed field region primarily confined to the center of the structure.

## Introduction

Coronal or helmet streamers are large-scale, relatively long-lived structures observed in the solar corona. They encompass spatial scales of a few solar radii, and exist on time scales of a few days to as long as a few months. Coronal streamers appear in the corona (in white light) as structures that are brighter than the background [e.g., Poland, 1978; Priest, 1982, and references therein] and have been generally interpreted as outlining coronal magnetic fields. As the white-light in the corona near the sun is predominantly scattered by electrons in the coronal plasma, the larger relative brightness of these objects indicates that they are denser than the background.

Streamers are often associated with coronal mass ejections (CMEs), and observations have shown specific instances where a streamer was disrupted by a CME [e.g., Hildner et al., 1975; Illing and Hundhausen, 1986]. Current theoretical ideas suggest that CMEs may result from the driven instability of large-scale plasma and magnetic field configurations [e.g., Priest, 1988]; therefore, the magnetic structure of coronal streamers may play a role in CME initiation.

One of the first numerical studies of coronal streamers was by Pneumann and Kopp [1971]. They used an iterative procedure to solve the steady-state MHD equations. Their solution stipulated that the plasma be hydrostatic in the closed field region, with flow parallel to the magnetic field in the open field region. A cusplike neutral point was assumed to occur at the boundary between the open and closed field lines. Using this model, Pneumann and Kopp obtained a two-dimensional (axisymmetric) streamerlike configuration. Their solution exhibited the general properties expected of a coronal streamer: a region of closed magnetic field, bounded by a current sheet and open magnetic fields.

Another approach that has been used to investigate the structure of coronal streamers is the time-dependent MHD simulation [e.g., Steinolfson et al., 1982; Washimi et al., 1987]. In this approach, one solves the MHD equations as an initial-value problem, starting from a given initial condition. The system is evolved in time until a steady-state (in practice only a quasi-steady) configuration is reached. This approach has the advantage that the only assumption imposed is the initial condition, and assumptions about the properties of the solution, such as the nature of the currents, etc., are unnecessary. Like the Pneumann and Kopp calculation, these earlier MHD simulations of streamerlike configurations were also 2-D (axisymmetry assumed). However, observations of coronal streamers [e.g., Poland, 1978] indicate that they have a finite longitudinal extent, and are thus 3-D objects. No models have attempted to depict the 3-D magnetic topology of a streamer. In this paper, we use time-dependent MHD simulations to examine, for the first time, an idealized, three-dimensional streamerlike configuration.

## Description of the Simulation

Our simulation code solves the following form of the MHD equations (in normalized units) in spherical coordinates using a two-step Lax-Wendroff method [similar to Linker et al., 1990]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{v}) = 0 \quad (1)$$

$$\frac{\partial}{\partial t} (\rho \bar{v}) + \nabla \cdot (\rho \bar{v} \bar{v} + P \mathbf{I}) = \bar{J} \times \bar{B} - \rho g \hat{r} / r^2 - \frac{1}{R_e} \nabla \cdot \mathbf{W} \quad (2)$$

$$\frac{\partial P}{\partial t} + \nabla \cdot (P \bar{v}) = (\gamma - 1) \left( -P (\nabla \cdot \bar{v}) + \eta J^2 - \frac{1}{R_e} \mathbf{W} : \nabla \bar{v} \right) \quad (3)$$

$$\frac{\partial \bar{B}}{\partial t} - \nabla \times (\bar{v} \times \bar{B}) = -\nabla \times (\eta \bar{J}) \quad (4)$$

In the above equations,  $\mathbf{W}$  is a viscous stress tensor used for numerical stability purposes.  $R_e$  is the fluid Reynolds number ( $\approx 50 - 100$ ),  $\eta$  is the resistivity, and  $\gamma$  is the polytropic index. The simulation grid is  $71 \times 17 \times 17$  (in  $r, \theta, \phi$ ) with  $\Delta r = 0.1$ ,  $\Delta \theta = 6^\circ$ , and  $\Delta \phi = 6^\circ$  (this includes extra grid points used in the  $\theta$  and  $\phi$  directions for formulating the boundary conditions). When expressed in spherical coordinates, Equations (1-4) have a singularity at the poles of the coordinate system (e.g.,  $\theta = 0^\circ$ ). We calculate at the poles by deriving and differencing a form of the equations valid at the poles [Linker, 1987].

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